AQA

A-LEVEL
Mathematics

MPC4 Pure Core 4
Mark scheme

6360

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
$\surd$ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}=\right) \quad 3(t-1)^2$	**B1**		ACF e.g. $3t^2-6t+3$
	$\left(\frac{\mathrm{d}y}{\mathrm{d}t}=\right) \quad 3+16t^{-3}$	**B1**		ACF
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3+16t^{-3}}{3(t-1)^2}$	**B1ft**	**3**	ACF: must see $\frac{\mathrm{d}y}{\mathrm{d}x}=\cdots$ but **ft** on their $\frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}$ **provided** numerator is of the form $3\pm kt^{-3}$ **and** denominator is a quadratic in *t*.
	Accept missing $\frac{\mathrm{d}x}{\mathrm{d}t}$ and/or $\frac{\mathrm{d}y}{\mathrm{d}t}$ or poor notation but must see $\frac{\mathrm{d}y}{\mathrm{d}x}=\ldots$ on final line If answer left as the product $\frac{1}{3(t-1)^2}\times 3+16t^{-3}$ with missing brackets then **B0(ft)** but can score the next **B1** in part (b).			
(b)	At $t=2$, $m=\frac{3+\frac{16}{8}}{3\times 1^2}=\frac{5}{3}$	**B1**		From a **correct** $\frac{\mathrm{dy}}{\mathrm{dx}}$. PI by next line.
	Gradient of normal $=-\frac{3}{5}$			
	(Normal is) $y-4=-\frac{3}{5}(x-1)$	**M1**		Use of their $-\frac{1}{m}$ with $x=1$ **and** $y=4$.
	$3x+5y-23=0$	**A1**	**3**	Integer coefficients with all terms on one side (in any order) and = 0 on the other.
	If $y=mx+c$ is used they must use $x=1$ and $y=4$ to find a value for c to earn the **M1** mark. An answer such as $0=-10y-6x+46$ would score **A1** but $5y=-3x+23$ or $3x+5y=23$ is **A0**.			
	Total		**6**	

<table>
<tr><th>Q2</th><th>Solution</th><th>Mark</th><th>Total</th><th>Comment</th></tr>
<tr><td>(a)</td><td>$R = \sqrt{58}$
$\sqrt{58}\cos\alpha = 7$ or $\sqrt{58}\sin\alpha = 3$ or $\tan\alpha = \frac{3}{7}$
$\alpha = 23.2^0$</td><td>B1
M1
A1</td><td>

3</td><td>Must see $R = \sqrt{58}$, $R = \pm\sqrt{58}$ is B0
ft on their value of R.
Must see $\alpha = 23.2^0$
Allow AWRT 22.9^0 to 23.3^0.</td></tr>
<tr><td></td><td colspan="4">Accept any decimal equivalent to $\sqrt{58}$ to at least 3 SF provided it is rounded correctly – e.g. 7.62, 7.616 etc.
e.g. using $R = 7.61$ to get $\alpha = 23.1^0$ would score B0 M1 A1.
Explicit use of $\cos\alpha = 7$ and $\sin\alpha = 3$ to get to $\tan\alpha = \frac{3}{7}$ is M0 A0 but marks in (b) are available.
Candidates who write $R\cos\alpha = 7$ and $R\sin\alpha = 3$ without finding R but reach a correct value for α score M1 A1.
An expression of the form $Rcos(x - \alpha)$ can score the B1 and/or A1 if R and/or α are correct.</td></tr>
<tr><td>(b)</td><td>$\cos^{-1}\left(\frac{5}{\sqrt{58}}\right)$ or 48.964 ...
and $311.0359\ldots$
36.1^0 and 167.1^0</td><td>M1
dM1
A1</td><td>

3</td><td>Finding an angle from $\cos^{-1}\left(\frac{5}{\mathrm{R}}\right)$.
For $360 - \cos^{-1}\left(\frac{5}{\mathrm{R}}\right)$ only between 0^0 and 360^0
CAO. These two values only.</td></tr>
<tr><td></td><td colspan="4">Question says ‘Use your answer to part (a)’ so using a different method or NMS is 0/3.
For M1 and dM1 marks accept 2 SF or better.
dM1 is for correct ft fourth quadrant solution (360^0 – first solution) and NO others between 0^0 and 360^0.
The dM1 mark could be PI if candidate goes straight to the two correct answers from the M1 mark.
Ignore any solutions outside the interval $0^0 \le \theta \le 180^0$ for final A1.
Condone omission of degree symbol or other letter in place of θ.</td></tr>
<tr><td></td><td>Total</td><td></td><td>6</td><td></td></tr>
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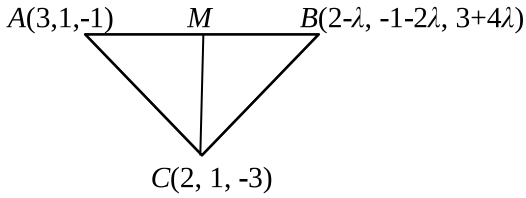
Q3	Solution	Mark	Total	Comment
(a)(i)	$\mathrm{f}\left(-\frac{2}{3}\right) = 6\left(-\frac{2}{3}\right)^3 - 11\left(-\frac{2}{3}\right)^2 + 2\left(-\frac{2}{3}\right) + 8$ $= 0$ (hence) factor	**M1** **A1**	 **2**	Attempt at $\mathrm{f}\left(-\frac{2}{3}\right)$ **Correct** arithmetic seen **and** conclusion.
	Question says 'Use the factor theorem' so long division scores **0/2**. Candidate could imply conclusion at beginning, e.g. $3x + 2$ is a factor if $\mathrm{f}\left(-\frac{2}{3}\right) = 0$ etc. Just $\mathrm{f}\left(-\frac{2}{3}\right) = 6\left(-\frac{2}{3}\right)^3 - 11\left(-\frac{2}{3}\right)^2 + 2\left(-\frac{2}{3}\right) + 8 = -8 + 8 = 0$ and conclusion is **M1 A0** as no 'arithmetic' but seeing such as $\mathrm{f}\left(-\frac{2}{3}\right) = -\frac{48}{27} - \frac{44}{9} - \frac{4}{3} + 8$ OE $= 0$ and conclusion would score **M1 A1.**			
(a)(ii)	Attempt at quadratic factor $2x^2 - 5x + 4$ $b^2 - 4ac = 25 - 32$ or -7 < 0 OE so no (more) factors / roots / solutions	**M1** **A1** **dM1** **A1**	 **4**	e.g. long division or factorising Correct quadratic Correct $b^2 - 4ac$ for **their** quadratic Valid reason and conclusion needed.
	To earn the **M1** for any approach we must see either $(2x^2 - 5x + c)$ or $(2x^2 + bx + 4)$ PI. If $\left(x + \frac{2}{3}\right)$ is used instead of $(3x + 2)$ we need $(6x^2 - 15x + c)$ or $(6x^2 + bx + 12)$ for **M1** The d**M1** is for a **correct** $b^2 - 4ac$ for **their** quadratic (can be unsimplified) – e.g. $5^2 - 4(2)(4)$. If using completing the square we need to see the form $p(x - q)^2 = r$ **correct** for their quadratic For final **A1**, candidates must have a **correct** quadratic, **correct** discriminant and **correct** conclusion.			
(b)	$\mathrm{g}(x) = (3x + 2)(2x^2 - 5x + 4)$ $-(3x + 2)(2x - 2)$ $= (3x + 2)(2x^2 - 7x + 6)$ $= (3x + 2)(x - 2)(2x - 3)$	 **M1** **A1**	 **2**	 or $\mathrm{g}(x) = 6x^3 - 17x^2 + 4x + 12$ Attempt at quadratic factor Correct three linear factors
	If using known factor **M1** could be earned for $(3x + 2)(2x^2 - 7x + c)$ or $(3x + 2)(2x^2 + bx + 6)$ If using another factor **M1** could be earned for $(x - 2)(6x^2 - 5x + c)$ or $(x - 2)(6x^2 + bx - 6)$ or $(2x - 3)(3x^2 - 4x + c)$ or $(2x - 3)(3x^2 + bx - 4)$ If a calculator is used to solve the cubic in order to factorise, it scores **0/2** or **2/2** e.g. $\left(x + \frac{2}{3}\right)(x - 2)\left(x - \frac{3}{2}\right)$ would score **0/2** but $6\left(x + \frac{2}{3}\right)(x - 2)\left(x - \frac{3}{2}\right)$ would score **2/2**.			
(c)	$\mathrm{h}(x) = \dfrac{\mathrm{g}(x)}{6x^3 - 5x^2 - 6x}$ $= \dfrac{(3x+2)(x-2)(2x-3)}{x(3x+2)(2x-3)}$ $\left(= \frac{x-2}{x}\right) \quad = 1 - 2x^{-1}$	 **M1** **A1**	 **2**	Attempt at full linear factors **and** cancelling at least one common factor. PI by correct answer in any form. Final answer must be seen in this form.
	For **M1** we need to see $\dfrac{\text{their three linear factors from (b)}}{x(3x+2)(rx+s)}$ cancelled down by at least one factor. No need to state $p = 1, q = -2$ and $n = -1$; apply **ISW** once correct answer seen.			
	Total		**10**	

Q4	Solution	Mark	Total	Comment
(a)	$(1-4x)^{-\frac{1}{2}} = 1 + 2x + kx^2$	**M1**		$k \neq 0$
	$= 1 + 2x + 6x^2$	**A1**	**2**	
(b)	$(16+4x)^{\frac{3}{4}} = 16^{\frac{3}{4}}\left(1+\frac{4x}{16}\right)^{\frac{3}{4}}$	**B1**		OE e.g. $8\left(1+\frac{x}{4}\right)^{\frac{3}{4}}$
	$\left(1+\frac{4}{16}x\right)^{\frac{3}{4}} = 1 + \frac{3}{4}.\left(\frac{4x}{16}\right) + \frac{3}{4}.-\frac{1}{4}\left(\frac{4x}{16}\right)^2.\frac{1}{2}\ \ldots$	**M1**		Condone missing / poor use of brackets
	$(16+4x)^{\frac{3}{4}} =$ $8\left(1+\frac{3}{16}x-\frac{3}{512}x^2\right)$ or $8+\frac{3}{2}x-\frac{3}{64}x^2$	**A1**	**3**	Must be 8 not $16^{3/4}$
	or $(16+4x)^{\frac{3}{4}} = 16^{\frac{3}{4}} + \frac{3}{4}(16)^{-\frac{1}{4}}(4x) + \frac{3}{4}.-\frac{1}{4}(16)^{-\frac{5}{4}}(4x)^2.\frac{1}{2}\ldots$ **M1** $= 8+\frac{3}{2}x-\frac{3}{64}x^2$ **A2**			
(c)	$(1+2x+6x^2)\left(8+\frac{3}{2}x-\frac{3}{64}x^2\right)$	**M1**		Setting up the product of their two expansions - be convinced
	$= 8 + \frac{35}{2}x + \frac{3261}{64}x^2$	**A1**	**2**	CAO
	Coefficients must be in **simplest** form or as full **exact** decimals $= 8 + 17.5x + 50.953125x^2$ Allow mixed numbers – e.g. $8 + 17\frac{1}{2}x + 50\frac{61}{64}x^2$. Allow terms in any order. **Ignore** terms in higher powers of x even if wrong.			
	Total		**7**	

Q5	Solution	Mark	Total	Comment
(a)	$\sin 3\theta = \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$	B1		**Use** of the **correct** sin $(A+B)$ identity. PI by next two B marks
	$= 2\sin\theta\cos\theta\cos\theta$	B1		**Use** of the **correct** sin *2A* identity
	$+ (1 - 2\sin^2\theta)\sin\theta$	B1		**Use** of a **correct** cos *2A* identity in ACF
	$= 3\sin\theta - 4\sin^3\theta$	B1	4	AG – be convinced
	For third **B1** they could use $\cos^2\theta - \sin^2\theta$ or $2\cos^2\theta - 1$ rather than $1 - 2\sin^2\theta$ for $\cos 2\theta$. Condone missing brackets **only** if recovered. For fourth **B1** we must see $\cos^2\theta = 1 - \sin^2\theta$ **used** to obtain the printed answer and no errors. Condone use of any other letter in place of θ for first 3 marks but withhold the final **B1**.			
(b)	From (a) $2\sin^3\theta = \frac{1}{2}(3\sin\theta - \sin 3\theta)$	B1		OE: ACF to enable them to replace $2\sin^3\theta$.
	$\int 2\sin^3\theta \, d\theta = p\cos\theta + q\cos 3\theta$	M1		p and q are any constants
	$= -\frac{3}{2}\cos\theta + \frac{1}{6}\cos 3\theta$	A1		Both integrated correctly
	$\int 3 \, d\theta = 3\theta$	B1		Must be 3θ
	$\int \quad = \left[-\frac{3}{2}\cos\frac{\pi}{6} + \frac{1}{6}\cos\frac{3\pi}{6} + 3\frac{\pi}{6}\right]$			
	$-\left[-\frac{3}{2}cos0 + \frac{1}{6}cos0 + (0)\right]$	dM1		$F\left(\frac{\pi}{6}\right) - F(0)$ **correct** for their integrated function - no MC allowed here.
	$= \left[-\frac{3\sqrt{3}}{4} + (0) + \frac{\pi}{2}\right] - \left[-\frac{3}{2} + \frac{1}{6} + (0)\right]$			
	$= -\frac{3\sqrt{3}}{4} + \frac{4}{3} + \frac{\pi}{2}$	A1	6	ACF but must be exact. Apply ISW (if necessary) after correct answer seen.
	The **dM1** and **A1** marks can be earned if candidate's 3θ term is 3x say. The **dM1** mark can be PI by correct final answer. With a common denominator, answer is $\frac{1}{12}\left(16 + 6\pi - 9\sqrt{3}\right)$. If hybrid use of x and θ in trig. Functions – contact your Team Leader. **NMS** is 0/6			
	Total		**10**	

Q6	Solution	Mark	Total	Comment
	NO MISREADS ARE ALLOWED IN THIS QUESTION			
(a)	$\begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ -2 \\ p \end{bmatrix}$ (From x or y) $\lambda = -1$ (Using z) $-1 = 3 - p \quad \rightarrow \quad p = 4$	M1 A1	2	$\lambda = -1$ seen or used AG ; must see $p = 4$ justfied
(b)	$(-1)(2) + (-2)(-3) + (4)(-1) = 0$ perpendicular / 90^0 OE	B1	1	**Correct** scalar product = 0 and perpendicular or 90^0 OE seen.
	Accept $-2 + 6 - 4 = 0$ for sufficient evidence of correct scalar product. Accept inclusion of λ and μ in their scalar product.			
(c)	$2 - \lambda = 2 + 2\mu$ $-1 - 2\lambda = 1 - 3\mu$ $3 + 4\lambda = -3 - \mu$	M1		Setting up at least two of these equations **and** attempt to solve for λ or μ.
	Solving x and y gives $\lambda = -\frac{4}{7}$ $\mu = \frac{2}{7}$	A1		Either correct.
	Checking in z gives $\frac{5}{7} \neq -\frac{23}{7}$ so l_1 and l_2 (or they) don't intersect / skew	E1	3	**Correct** use of **correct** λ and μ to show they don't satisfy the unused equation and a conclusion
	Solving x and z gives $\lambda = -\frac{12}{7}$ $\mu = \frac{6}{7}$ and checking in y gives $\frac{17}{7} \neq -\frac{11}{7}$. Solving y and z gives $\lambda = -\frac{10}{7}$ $\mu = -\frac{2}{7}$ and checking in x gives $\frac{24}{7} \neq \frac{10}{7}$. For **E1** they could correctly use **correct** value of λ or μ in third equation and show inconsistent value for μ or λ (or that the point of intersection is not consistent) **and** state a conclusion.			
(d)	$AC^2 = 5$ or $AC = \sqrt{5}$	B1		Condone any wrong signs seen in $\overrightarrow{\boldsymbol{AC}}$ PI if seen correct on **M1** line.
	$(\overrightarrow{\boldsymbol{BC}} =)$ $\begin{bmatrix} \lambda \\ 2 + 2\lambda \\ -6 - 4\lambda \end{bmatrix}$	B1		Accept $+/-$ this vector PI if seen correct on **M1** line.
	$\lambda^2 + (2 + 2\lambda)^2 + (-6 - 4\lambda)^2 = 5$	M1		Forming a quadratic equation in λ from $BC^2 = AC^2$ (ft on BC and AC.) No square root signs.
	$21\lambda^2 + 56\lambda + 35 \quad (= 0)$	A1		OE. Withhold if a component error in $\overrightarrow{\boldsymbol{BC}}$
	$(\lambda + 1)(3\lambda + 5) \quad (= 0)$	dM1		OE. Attempt at factors for a three term quadratic PI by correct λ values
	$\left(\frac{11}{3}, \frac{7}{3}, -\frac{11}{3}\right)$ uniquely identified as B	A1	6	Accept as a column vector Withhold if a component error in $\overrightarrow{\boldsymbol{BC}}$**.**
	For **dM1** the factors must give their $3\lambda^2$ and $+5$ terms or the formula must be used correctly.			
	Total		**12**	

Q6	Solution	Mark	Total	Comment
(d)	**Alternative solutions**  			
	<u>Use of scalar product</u> M $(2-\lambda, -1-2\lambda, 3+4\lambda)$ (**B1**) $\overrightarrow{\boldsymbol{CM}} = \underline{\boldsymbol{m}} - \underline{\boldsymbol{c}} = \begin{bmatrix}-\lambda\\-2-2\lambda\\6+4\lambda\end{bmatrix}$ (**B1**) $\overrightarrow{\boldsymbol{CM}}\cdot\begin{bmatrix}-1\\-2\\4\end{bmatrix}=0$ $(-\lambda)(-1)+(-2-2\lambda)(-2)+(6+4\lambda)(4)=0$ (**M1**) $\lambda+4+4\lambda+24+16\lambda=0$ (**A1**) a correct linear equation in λ $21\lambda+28=0$ $\rightarrow \lambda=-\frac{4}{3}$ (**dM1**) solving for λ $\rightarrow$ $M\left(\frac{10}{3},\frac{5}{3},-\frac{7}{3}\right)$ so B is $\left(\frac{11}{3},\frac{7}{3},-\frac{11}{3}\right)$ (**A1**) accept as column vector			
	<u>Use of Pythagoras</u> $AC^2=5$ (or $AC=\sqrt{5}$) (**B1**) M is $(2-\lambda,-1-2\lambda,3+4\lambda)$ $\overrightarrow{\boldsymbol{AM}}=\begin{bmatrix}-1-\lambda\\-2-2\lambda\\4+4\lambda\end{bmatrix}$ **or** $\overrightarrow{\boldsymbol{CM}}=\begin{bmatrix}-\lambda\\-2-2\lambda\\6+4\lambda\end{bmatrix}$ (**B1**) $AM^2 \quad + \quad CM^2 \quad = \quad AC^2$ $(-1-\lambda)^2+(-2-2\lambda)^2+(4+4\lambda)^2 + (-\lambda)^2+(-2-2\lambda)^2+(6+4\lambda)^2=(-1)^2+0^2+(-2)^2$ (**M1**) $k(3\lambda^2+7\lambda+4)=0$ (**A1**) ($\lambda=-1$) or $\lambda=-\frac{4}{3}$ (**dM1**) $B\left(\frac{11}{3},\frac{7}{3},-\frac{11}{3}\right)$ (**A1**) Accept as column vector **Withhold the A marks if there are any earlier error(s) e.g.** $\overrightarrow{\boldsymbol{CM}}=\begin{bmatrix}\lambda\\-2-2\lambda\\6+4\lambda\end{bmatrix}$ **etc.**			
	Could use the fact that M is the mid-point of AB in either of the above and express it as $M\left(\frac{5-\lambda}{2},-\lambda,1+2\lambda\right)$.			
	<u>Use of equal angles at A and B</u> $\overrightarrow{\boldsymbol{AB}}=\begin{bmatrix}-1\\-2\\4\end{bmatrix}$ (direction of l_1) **and** $\overrightarrow{\boldsymbol{AC}}=\begin{bmatrix}-1\\0\\-2\end{bmatrix}$ (**B1**) $\overrightarrow{\boldsymbol{BC}}=\begin{bmatrix}\lambda\\2+2\lambda\\-6-4\lambda\end{bmatrix}$ (**B1**) $\overrightarrow{\boldsymbol{AB}}\,.\,\overrightarrow{\boldsymbol{AC}}=\overrightarrow{\boldsymbol{BA}}\,.\,\overrightarrow{\boldsymbol{BC}}$ $\rightarrow(-1)(-1)+(-2)(0)+(4)(-2)=(1)(\lambda)+(2)(2+2\lambda)+(-4)(-6-4\lambda)$ (**M1**) $\rightarrow 1+0-8=\lambda+4+4\lambda+24+16\lambda$ (**A1**) any correct equation $(-7=21\lambda+28)$ $\rightarrow \lambda=-\frac{5}{3}$ (**dM1**) solving for λ $B\left(\frac{11}{3},\frac{7}{3},-\frac{11}{3}\right)$ (**A1**)			
	Correct $\overrightarrow{\boldsymbol{BA}},\overrightarrow{\boldsymbol{CA}}$ and/or $\overrightarrow{\boldsymbol{CB}}$ could score the **B1** marks. Allow $\pm\overrightarrow{\boldsymbol{AB}}\,.\,\overrightarrow{\boldsymbol{AC}}=\pm\overrightarrow{\boldsymbol{BA}}\,.\,\overrightarrow{\boldsymbol{BC}}$ for the **M1** mark but must be 4 vectors containing angles at A and B..			
	Use of equivalent direction vectors always possible. If you come across a solution that you think falls into one of these methods and you're not sure how to mark it, please contact your Team Leader.			

Q7	Solution	Mark	Total	Comment
(a)	$\sin 3y + 3\mathrm{e}^{-2x}y + 2x^2 = 5$			
	$\frac{\mathrm{d}}{\mathrm{d}x}(\sin 3y) = 3\cos 3y \frac{\mathrm{d}y}{\mathrm{d}x}$	**B1**		
	$\frac{\mathrm{d}}{\mathrm{d}x}(3\mathrm{e}^{-2x}y) = p\mathrm{e}^{-2x}y + q\mathrm{e}^{-2x}\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1**		
	$= -6\mathrm{e}^{-2x}y + 3\mathrm{e}^{-2x}\frac{\mathrm{d}y}{\mathrm{d}x}$	**A1**		
	$\frac{\mathrm{d}}{\mathrm{d}x}(2x^2 = 5)$ gives $+4x = 0$	**B1**		Both correct, looking for $+4x = 0$.
	$\frac{\mathrm{d}y}{\mathrm{d}x}(3\cos 3y + 3\mathrm{e}^{-2x}) - 6\mathrm{e}^{-2x}y + 4x = 0$	**M1**		Attempt to factor out $\frac{\mathrm{d}y}{\mathrm{d}x}$ from an expression involving exactly **two** $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms. PI by next line.
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6\mathrm{e}^{-2x}y - 4\mathrm{x}}{3\cos 3y + 3\mathrm{e}^{-2x}}$	**A1**	**6**	Can be earned without scoring second **B1**.
	Diff. w.r.to y marks as $3cos3y$ (**B1**) + $(3e^{-2x} - 6ye^{-2x}\frac{dx}{dy})$ (**M1 A1**) + $4x\frac{dx}{dy} = 0$ (**B1**) then (**M1 A1**) Ignore spurious $\frac{\mathrm{d}y}{\mathrm{d}x} = \cdots$ for first four marks. Penalise with **M0 A0** unless recovered. Candidates who miss out $= 0$ will lose the second **B** mark but can still earn the final **A** mark if recovered.			
(b)(i)	At $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ $\quad 6\mathrm{e}^{-2x}y - 4x = 0$	**M1**		Putting the numerator of their $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and attempting to solve for y.
	$y = \frac{2}{3}x\,\mathrm{e}^{2x}$	**A1**	**2**	OE any **exact** value for $\frac{2}{3}$ including $0.\dot{6}$
	To score the **A1** mark in (b)(i) their $\frac{\mathrm{d}y}{\mathrm{d}x}$ in part (a) **must** be **correct** but allow the marks in (b)(ii).			
(b)(ii)	Using $y = \frac{2}{3}x\,\mathrm{e}^{2x}$ in original equation gives			
	$\sin\left(3 \times \frac{2}{3}x\,\mathrm{e}^{2x}\right) + 3\mathrm{e}^{-2x} \times \frac{2}{3}x\,\mathrm{e}^{2x} + 2x^2 = 5$	**M1**		Attempt to substitute their expression of the form $y = rx\,\mathrm{e}^{2x}$ into original equation.
	$\sin(2x\,\mathrm{e}^{2x}) + 2x + 2x^2 = 5$	**A1**		**Correct** but need not be simplified
	$\mathrm{f}(x) = \sin(2x\,\mathrm{e}^{2x}) + 2x + 2x^2 - 5 = 0$			Rearrange into the form $\mathrm{f}(x) = 0$.
	$\left.\begin{array}{l}(\mathrm{f}(1) = 0.80\ldots + 2 + 2 - 5 =) \quad -0.198\ldots \\ (\mathrm{f}(1.2) = 0.96\ldots + 2.4 + 2.88 - 5 =) \quad 1.249\ldots\end{array}\right\}$	**dM1**		f(1) **and** f(1.2) correct to at least 1 SF.
	Sign change so $1 < x < 1.2$	**A1**	**4**	f(1) and f(1.2) **correct** and sign change implies conclusion.
	Leaving as $\sin(2x\,\mathrm{e}^{2x}) + 2x + 2x^2 = 5$ and using $x = 1$ and $x = 1.2$ to get 4.8 ... and 6.2… for LHS (**dM1**) and $4.8 < 5$ and $6.2 > 5$ OE with conclusion scores (**A1**).			
	Total		**12**	

Q8	Solution	Mark	Total	Comment
(a)	$A\left(\frac{1}{x}+\frac{1}{k-x}\right)=A\left(\frac{k-x+x}{x(k-x)}\right)\ \left(=A\left(\frac{k}{x(k-x)}\right)\right)$	**M1**		To compare with $\frac{1}{x(k-x)}$
	Comparing gives $Ak=1$ so $A=\frac{1}{k}$	**A1**	**2**	**NMS** $A=\frac{1}{k}$ scores 2/2
	Alternative: $\frac{1}{x(k-x)}=\frac{A}{x}+\frac{A}{k-x}\ \rightarrow 1=A(k-x)+Ax$ **(M1)** $\rightarrow\ 1=Ak\ \rightarrow\ A=\frac{1}{k}$ (**A1)**			
(b)	$\frac{1}{x(1200-x)}=\lambda\ \left(\frac{1}{x}+\frac{1}{1200-x}\right)$ $\lambda\int\frac{1}{x}+\frac{1}{1200-x}\,\mathrm{d}x\quad\left(=\frac{1}{3600}\int\mathrm{d}t\right)$	**M1**		**S**eparation of x terms into two fractions. Common multiple needn't be correct.
	$\lambda\ (\ln x-\ln(1200-x))$ $=\frac{t}{3600}\quad(+C)$	**A1 A1**		Correct log integrations (LHS)
	$\frac{1}{1200}(\ln 300-\ln 900)=C\ \rightarrow\ C=\frac{1}{1200}\ln\left(\frac{1}{3}\right)$	**dM1**		Using x = 300 and t = 0 to find a value for C
	$t=3\ln\left(\frac{3x}{1200-x}\right)$	**A1**	**5**	AG be convinced
	AG!!! - **must** see some evidence of **correct** log manipulation before awarding the final **A1** e.g. $\ln x-\ln(1200-x)=\frac{t}{3}+\ln\left(\frac{1}{3}\right)\rightarrow\ln x-\ln(1200-x)=\frac{t}{3}+\ln\left(\frac{1}{3}\right)\rightarrow\ln\left(\frac{3x}{1200-x}\right)=\frac{t}{3}$ to answer **Alternative Method** $\frac{dx}{dt}=\frac{x(1200-x)}{3600}\ \rightarrow\ \frac{dt}{dx}=\frac{3600}{x(1200-x)}\ \rightarrow\ \frac{dt}{dx}=3\left(\frac{1}{x}+\frac{1}{1200-x}\right)$ **M1** (using (a)) $\rightarrow t=3\ln x$ **A1** $-\,3\ln(1200-x)$ **A1** (+C) then as above for **dM1 A1**			
(c)(i)	Using $x=600\quad t=3\ln\left(\frac{1800}{1200-600}\right)$ OE	**M1**		e.g. $t=3\ln 3$ or 3.29…
	14.20 or 2.20(p.m.)	**A1**	**2**	**NMS**: correct time scores **2/2**
(c)(ii)	$t=3\ln\left(\frac{3x}{1200-x}\right)\ \rightarrow\ \frac{3x}{1200-x}=\mathrm{e}^{\frac{t}{3}}$	**M1**		OE for RHS - e.g. $\sqrt[3]{e^t}$
	$x=\frac{1200\mathrm{e}^{\frac{t}{3}}}{3+\mathrm{e}^{\frac{t}{3}}}$	**A1**		
	(when t = 4) $x=670$	**B1**	**3**	CAO ; **not** 670.094…
	OE could also include $\mathrm{e}^{t}=\left(\frac{3x}{1200-x}\right)^3$ etc. It is possible to score **M1 A0 B1.**			
	Total	**12**		