# Level 2 Certificate
# FURTHER MATHEMATICS
# 8365/1

Paper 1 Non-Calculator

**Mark scheme**

June 2021

Version: 1.0 Final

2 1 6 G 8 3 6 5 / 1 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

# Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

| | |
|---|---|
| **M** | Method marks are awarded for a correct method which could lead to a correct answer. |
| **M dep** | A method mark dependent on a previous method mark being awarded. |
| **A** | Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied. |
| **B** | Marks awarded independent of method. |
| **B dep** | A mark that can only be awarded if a previous independent mark has been awarded. |
| **ft** | Follow through marks. Marks awarded following a mistake in an earlier step. |
| **SC** | Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth. |
| **oe** | Or equivalent. Accept answers that are equivalent.<br>eg accept 0.5 as well as $\frac{1}{2}$ |
| **[a, b]** | Accept values between a and b inclusive. |
| **3.14…** | Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416 |

Examiners should consistently apply the following principles.

**Diagrams**

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

**Responses which appear to come from incorrect methods**

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

**Questions which ask candidates to show working**

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

**Questions which do not ask candidates to show working**

As a general principle, a correct response is awarded full marks.

**Misread or miscopy**

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

**Further work**

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

**Choice**

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

**Work not replaced**

Erased or crossed out work that is still legible should be marked.

**Work replaced**

Erased or crossed out work that has been replaced is not awarded marks.

**Premature approximation**

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

**Continental notation**

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 1 | $(5 - -3)^2 + (1 - 7)^2$ or $8^2 + (-6)^2$ | M1 | oe eg $8^2 + 6^2$ or $\sqrt{100}$ |
| | 10 | A1 | |
| | **Additional Guidance** | | |
| | | | |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 2 | $2x^5 - 7x^4$ | M1 | |
| | $10x^4$ or $(-)\ 28x^3$ | M1 | oe eg $5 \times 2x^{5-1}$ |
| | $\left(\frac{dy}{dx} =\right) 10x^4 - 28x^3$ with no additional terms | A1 | do not award for $y =$<br>or $\frac{d^2y}{dx^2} =$ on the answer line<br>SC2 $2x^4 - 7x^3 + 8x^4 - 21x^3$<br>SC1 $2x^4 - 7x^3 + x(8x^3 - 21x^2)$ |
| | **Additional Guidance** | | |
| | Allow $y =$ ….. for M marks but must be recovered for A1<br>$\left(\frac{dy}{dx} =\right) 10x^4 - 28x^3 + c$ | | M2A0 |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 3 | B | B1 | |
| | **Additional Guidance** | | |
| | | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 4(a) | $6 < x < 8$<br>or<br>$x < 8$ and $x > 6$ | B1 | oe eg $8 > x > 6$<br>word 'and' must be included if writing two inequalities |
| | **Additional Guidance** | | |
| | [6, 8] | | B1 |
| | Condone eg $x = 6 < x < 8$ | | B1 |
| | Incorrect inequality symbol eg $6 \leqslant x < 8$ or $6 < x > 8$ | | B0 |
| | $6 < f(x) < 8$ | | B0 |
| | B1 response with a list of integers on answer line | | B0 |
| | Only a list of integers | | B0 |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 4(b) | $-4 \leqslant g(x) < 5$<br>or<br>$g(x) < 5$ and $g(x) \geqslant -4$ | B2 | oe eg $5 > g(x) \geqslant -4$<br>word 'and' must be included if writing two inequalities for B2 or B1 or SC1<br>B1 $-4 < g(x) < 5$ or $-4 < g(x) \leqslant 5$<br>or $-4 \leqslant g(x) \leqslant 5$<br>or $g(x) < 5$ and $g(x) > -4$<br>or $g(x) \leqslant 5$ and $g(x) > -4$<br>or $g(x) \leqslant 5$ and $g(x) \geqslant -4$<br>or $k < g(x) < 5$ where k is less than 5<br>or $k \leqslant g(x) < 5$ where k is less than 5<br>or $-4 \leqslant g(x) < m$ where m is greater than $-4$<br>SC1 $-4 \leqslant x < 5$<br>or $x < 5$ and $x \geqslant -4$<br>or only $-4$ and 5 seen (condone 9 given as a range in this case) |

| Additional Guidance | |
|---|---|
| Condone $g(x)$ replaced by eg $y$ or g or $gx$ or f or $fx$ or G or $Gx$ or $x^2 - 4$<br>eg1 $-4 \leqslant f(x) < 5$<br>eg2 $-4 \leqslant y \leqslant 5$ | <br>B2<br>B1 |
| $[-4, 5)$ | B2 |
| $(-4, 5)$ or $(-4, 5]$ or $[-4, 5]$ | B1 |
| Condone eg $g(x) = -4 \leqslant g(x) < 5$ | B2 |
| Condone eg $g(x) = -4 < g(x) < 5$ | B1 |
| B2 response with a list of integers on answer line | B1 |
| B1 response with a list of integers on answer line | B0 |
| Only a list of integers | B0 |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 4(c) | **Alternative method 1** | | |
| | $x = 2h(x) - 3$ or $x = 2y - 3$ | M1 | oe |
| | $2x - 3$ | A1 | |
| | **Alternative method 2** | | |
| | $x = \frac{3 + h^{-1}(x)}{2}$ or $x = \frac{3 + y}{2}$ | M1 | oe |
| | $2x - 3$ | A1 | |
| | **Additional Guidance** | | |
| | Answer left as $y = 2x - 3$ | | M1A0 |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 5(a) | $2n + 47 = 5(n + 1)$<br>or<br>$2n + 47 = 5n + 5$ | M1 | oe equation with fraction eliminated |
| | 14 | A1 | |
| | **Additional Guidance** | | |
| | $n = 14$ from trial and error | | M1A1 |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 5(b) | 2 | B1 | |
| | **Additional Guidance** | | |
| | Do not allow $n \rightarrow 2$ | | |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 6 | 40 and 140 with no other values | B2 | B1 40 or 140 |
| | **Additional Guidance** | | |
| | sin 40 and sin 140 with no other incorrect answers | B1 | |
| | 40 and 140 but with other values (incorrect or outside range) | B1 | |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 7 | $2x^2 - x - 3$ or $2x^2 - 3x + 2x - 3$ | M1 | |
| | $4 > -x - 3$ | M1dep | oe eg $7 > -x$ |
| | $x > -7$ or $-7 < x$ | A1 | |
| | **Additional Guidance** | | |
| | = used instead of > throughout and not recovered on answer line | M2A0 | |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 8 | **Alternative method 1** | | |
| | $\sqrt{225} + \sqrt{144}$<br>or<br>15 + 12 | M1 | |
| | 27 | A1 | |
| | **Alternative method 2** | | |
| | $5\sqrt{3} + 4\sqrt{3}$ or $9\sqrt{3}$<br>or<br>9 x 3<br>or<br>15 + 12 | M1 | |
| | 27 | A1 | |
| | **Additional Guidance** | | |
| | | | |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 9 | Expanding second and third bracket<br>$3x^2 + 6x – 4x – 8$ with at least three terms correct<br>or<br>$3x^2 + 2x + k$ where $k$ is a non-zero constant | M1 | oe expansion attempt of one pair of brackets<br>eg1 first and second brackets<br>$6x^2 – 8x – 15x + 20$ with at least three terms correct<br>or<br>$6x^2 – 23x + k$ where $k$ is a non-zero constant<br>eg2 first and third brackets<br>$2x^2 + 4x – 5x – 10$ with at least three terms correct<br>or<br>$(2x^2 – x + k)$ where $k$ is a non-zero constant |
| | $6x^3 + 12x^2 – 8x^2 – 16x$<br>or $6x^3 + 4x^2 – 16x$<br>or $–15x^2 – 30x + 20x + 40$<br>or $–15x^2 – 10x + 40$ | M1dep | attempt at a full expansion with correct multiplication of their 3 or 4 terms by one of the terms in the remaining bracket<br>oe eg1 $6x^3 – 8x^2 – 15x^2 + 20x$<br>or $6x^3 – 23x^2 + 20x$<br>or $12x^2 – 16x – 30x + 40$<br>or $12x^2 – 46x + 40$<br>eg2 $6x^3 + 12x^2 – 15x^2 – 30x$<br>or $6x^3 – 3x^2 – 30x$<br>or $– 8x^2 – 16x + 20x + 40$<br>or $– 8x^2 + 4x + 40$ |
| | $6x^3 – 11x^2 – 26x + 40$ | A1 | fully correct simplified expansion<br>allow terms written in a different order |
| | **Additional Guidance** | | |
| | For the M marks terms may be seen in a grid | | |
| | First M1 do not allow any omissions or extra terms<br>eg1 $3x^2 + 6x – 8$<br>eg2 $3x^2 + 6x – 4x – 8 + x^2$ | <br>M0<br>M0 |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 10 | **Alternative method 1** | | |
| | (Second differences =) $-2$<br>or $-n^2$ | M1 | second differences seen at least once and not contradicted<br>may be seen by the sequence |
| | $0--1\ \ 1--4\ \ 0--9\ \ (-3--16)$<br>or 1 5 9 (13)<br>or<br>$-1-0\ \ -4-1\ \ -9-0\ \ (-16--3)$<br>or –1 –5 – 9 (–13) | M1dep | subtracts $-n^2$ from the given terms<br>or<br>subtracts the given terms from $-n^2$ |
| | $-n^2+4n-3$ | A1 | oe eg $4n-3-n^2$ |
| | **Alternative method 2** | | |
| | Any three of<br>$a+b+c=0$<br>$4a+2b+c=1$<br>$9a+3b+c=0$<br>$16a+4b+c=-3$ | M1 | using nth term = $an^2+bn+c$ |
| | $3a+b=1$<br>and $5a+b=-1$<br>or<br>$a=-1$ and $b=4$ | M1dep | oe<br>obtains two equations in the same two variables |
| | $-n^2+4n-3$ | A1 | oe eg $4n-3-n^2$ |

**Mark scheme and Additional Guidance continue on the next page**

| | | | |
|---|---|---|---|
| **10 cont** | **Alternative method 3** | | |
| | (Second differences =) $-2$<br>or $-n^2$ | M1 | second differences seen at least once and not contradicted<br>may be seen by the sequence |
| | $3a + b = 1$<br>and substitutes $a = -1$ | M1dep | oe eg $-3 + b = 1$ or $b = 4$ |
| | $-n^2 + 4n - 3$ | A1 | oe eg $4n - 3 - n^2$ |
| | **Additional Guidance** | | |
| | Condone use of $U_n$ | | M2A1 |
| | Condone working in different variable(s) eg $-n^2 + 4x - 3$ | | M2A1 |
| | Answer $-n^2$ … scores at least M1 | | |
| | Condone $-n^2 + 4n - 3 = 0$ or $n = -n^2 + 4n - 3$ | | M2A1 |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 11 | $\begin{bmatrix} 2a & 2b+0.4 \\ 0 & 1.2 \end{bmatrix}$<br>or $2a = k$ or $k = 1.2$<br>or $2b + 0.4 = 0$ | M1 | oe<br>any 3 terms correct in correct position<br>could be implied from second M mark |
| | $2a = k$<br>and<br>$2b + 0.4 = 0$ | M1dep | oe eg $2a = 1.2$ and $2b + 0.4 = 0$ |
| | $a = 0.6$ or $b = –0.2$ | M1 | oe |
| | $a = 0.6$ and $b = –0.2$ | A1 | oe |
| | **Additional Guidance** | | |
| | | | |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 12(a) | $(x-4)^2+(y+2)^2=20$ | B2 | B1 $(x-4)^2+(y+2)^2$ or 20 |
| | **Additional Guidance** | | |
| | $(x+4)^2+(y-2)^2=20$<br>$(x-4)^2+(y+2)^2=4^2+(-2)^2$<br>$(x-4)^2+(y+2)^2=\sqrt{20}$<br>$(x-4)^2-(y+2)^2=20$<br>$(x-4)^2+(y--2)^2=20$<br>$(x-4)^2+(y--2)^2=(\sqrt{20})^2$<br>ignore further working | | B1<br>B1<br>B1<br>B1<br>B2<br>B2 |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 12(b) | (Gradient AC =) $\frac{0--2}{8-4}$ or $\frac{2}{4}$ | M1 | oe |
| | (Gradient of tangent =) negative reciprocal of their $\frac{2}{4}$<br>or –2 | M1 | oe<br>ft their gradient AC<br>only gradient –2 seen is M2 |
| | $y=-2x+16$ | A1 | oe |
| | **Additional Guidance** | | |
| | It is possible to find an incorrect gradient of AC and then get the second M mark for finding the negative reciprocal of this | | M0M1A0 |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 13(a) | $k^2 = \frac{49}{16}$ or $k = \sqrt{\frac{49}{16}}$ | M1 | oe |
| | $\frac{7}{4}$ | A1 | oe |
| | **Additional Guidance** | | |
| | $-\frac{7}{4}$ or $\pm\frac{7}{4}$ will not gain the A mark unless recovered | | M1A0 |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 13(b) | $\frac{4}{7}$ | B1ft | oe allow decimal rounded to 2dp or better ft their (a) but not if $k = 0$ or 1 |
| | **Additional Guidance** | | |
| | $\frac{1}{\frac{4}{7}}$ | | B0 |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 14 | **Alternative method 1** Eliminates $b$ from first two equations before eliminating a second variable | | |
| | Correct attempt to eliminate $b$ from LHS of first two equations | M1 | eg $2(4a - b + 3c) + 3a + 2b - c$<br>or $11a + 5c$<br>adding or subtracting the two equations can be implied from two terms correct |
| | Correct attempt to eliminate $a$ or $c$ from LHS of third equation and their equation in $a$ and $c$ | M1dep | eg $11a + 5c + 2a - 5c$<br>or $2(11a + 5c) - 11(2a - 5c)$ |
| | Correct equation in $a$ or $c$ | M1dep | eg $13a = 52$ or $65c = 195$<br>implied by $a = 4$ or $c = 3$ with M2 |
| | Two correct values with M3 | A1 | eg $a = 4$ and $c = 3$ with M3 |
| | $a = 4$ and $b = -2$ and $c = 3$<br>with M3 | A1 | |
| | **Alternative method 2** Eliminates $a$ or $c$ before eliminating a second variable | | |
| | Two correct attempts to eliminate the same variable ($a$ or $c$) from LHS | M1 | eg (eliminating $a$)<br>$4a - b + 3c - 2(2a - 5c)$<br>and $2(3a + 2b - c) - 3(2a - 5c)$<br>or<br>$-b + 13c$ and $4b + 13c$ |
| | Correct attempt to eliminate a second variable from LHS of their two equations | M1dep | eg $-b + 13c - (4b + 13c)$ |
| | Correct equation in one variable | M1dep | eg $-5b = 10$<br>implied by $b = -2$ with M2 |
| | Two correct values with M3 | A1 | eg $b = -2$ and $a = 4$ with M3<br>or<br>$b = -2$ and $c = 3$ with M3 |
| | $a = 4$ and $b = -2$ and $c = 3$<br>with M3 | A1 | |

**Additional Guidance is on the next page**

| | Additional Guidance | |
|---|---|---|
| **14 cont** | For the first two marks ignore the RHS of the equations | |
| | First two method marks may be seen in one attempt<br>eg Alt1 $2(4a - b + 3c) + 3a + 2b - c + 2a - 5c$ | M1M1 |
| | Elimination may be seen from other approaches<br>eg1 Alt 1 (equates expressions for $2b$ from first two equations)<br>$2(4a + 3c - 27) = 5 - 3a + c$<br>eg2 Alt 2 (rearranges third equation to $a = 2.5c - 3.5$ and substitutes into first two equations)<br>$4(2.5c - 3.5) - b + 3c$ and $3(2.5c - 3.5) + 2b - c$ | M1<br><br>M1 |
| | Correct values with no working | M0A0 |

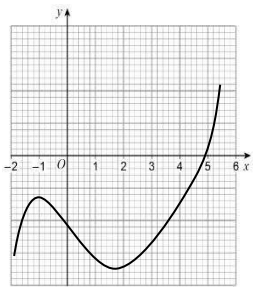
| Question | Answer | Mark | Comments |
|---|---|---|---|
| 15 | $\tan x = (\pm)\frac{1}{\sqrt{3}}$ or $\tan x = (\pm)\frac{\sqrt{3}}{3}$ | M1 | |
| | 30 with no incorrect solutions within the given range | A1 | ignore correct solutions outside the given range. |
| | **Additional Guidance** | | |
| | | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 16(a) | $200\left(-\frac{1}{2}\right)^3 + 100\left(-\frac{1}{2}\right)^2 - 18\left(-\frac{1}{2}\right) - 9$ | M1 | oe<br>eg $200\left(-\frac{1}{8}\right) + 100\left(\frac{1}{4}\right) - 18\left(-\frac{1}{2}\right) - 9$ |
| | $-25 + 25 + 9 - 9 = 0$ with M1 seen | A1 | must evaluate each term and equate to zero |
| | **Additional Guidance** | | |
| | Condone $\left(\frac{1}{2}\right)^2$ for $\left(-\frac{1}{2}\right)^2$ | | |
| | $200\left(-\frac{1}{2}\right)^3 + 100\left(-\frac{1}{2}\right)^2 - 18\left(-\frac{1}{2}\right) - 9 = 0$ | | M1A0 |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 16(b) | $(100x^2 - 9)$ | M1 | |
| | $(10x - 3)(10x + 3)$ or $(x = )\sqrt{\frac{9}{100}}$ | M1dep | oe eg $(x = )\sqrt{0.09}$ |
| | –0.5 and –0.3 and 0.3 | A1 | oe eg fractions |
| | **Additional Guidance** | | |
| | –0.5 and –0.3 or –0.5 and 0.3 with the other solution missing implies $(100x^2 - 9)$ | | M1M0A0 |
| | –0.3 and 0.3 on answer line implies $(10x - 3)(10x + 3)$ | | M2A0 |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 17 | $x - 4$ or $4 - x$ seen in working | M1 | from a subtraction of the quadratic and linear |
| | $y = x - 4$ drawn | A1 | |
| | 5.3 and 1.7 and $y = x - 4$ drawn | A1 | Allow [5.2, 5.4] and [1.6,1.8] |
| | **Additional Guidance** | | |
| | Solutions with correct graph not seen eg from formula | | M0A0A0 |
| | Solutions from quadratic graph drawn | | M0A0A0 |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 18 | $7^2 = x^2 + 3^2 - 2 \times 3 \times x \cos 60°$ | M1 | oe |
| | $x^2 - 3x - 40 \ (= 0)$ | A1 | |
| | $(x - 8)(x + 5) \ (= 0)$<br>or<br>$\frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times -40}}{2 \times 1}$ | M1 | oe<br>follow through their three term quadratic |
| | 8 | A1 | |
| | **Additional Guidance** | | |
| | If –5 is also given as an answer then do not award final A mark | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 19 | Cubic curve from $x = -2$ to $x = 6$<br>and<br>maximum point at $(-1, a)$<br>where $a$ is negative<br>and<br>minimum point at $(2, b)$<br>where $b$ is less than $a$<br>and<br>increasing through (5, 0) | B4 | B3 curve from $x = -2$ to $x = 6$<br>and<br>maximum point at $(-1, c)$<br>where $c$ is any value<br>and<br>minimum point at $(2, d)$<br>where $d$ is less than $c$ and $d$ is negative<br>and<br>increasing through (5, 0)<br>or<br>a B4 response apart from cubic curve not drawn from $x = -2$ to $x = 6$<br><br>B2 curve with maximum point at $(-1, e)$<br>where $e$ is any value<br>and<br>minimum point at $(2, f)$<br>where $f$ is less than $e$<br><br>B1 curve with maximum point at $(-1, g)$<br>where $g$ is negative<br>or<br>curve with minimum point at $(2, h)$<br>where $h$ is negative<br>or<br>curve increasing through (5, 0)<br>SC2 max and min correct and increasing through (5, 0) but with straight lines rather than a curve. |
| | **Additional Guidance** | | |
| |  | | B4 |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 20 | 5 × … | M1 | oe eg listing the 5 possible first digits |
| | 5 × 5 × 4 × 3 | M1dep | |
| | 300 | A1 | SC1 (6 × 5 × 4 × 3 =) 360 |
| | **Additional Guidance** | | |
| | | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 21 | **Alternative method 1** Works out $\frac{1}{2} \times (6 + 2\sqrt{7})$ | | |
| | $\frac{1}{2} \times (6 + 2\sqrt{7}) \times AD$ | M1 | oe eg $(3 + \sqrt{7}) \times AD$<br>or $(3 + \sqrt{7}) \times AC \sin C$<br>may be implied |
| | $\frac{13 + 3\sqrt{7}}{3 + \sqrt{7}} \times \frac{3 - \sqrt{7}}{3 - \sqrt{7}}$ | M1 | ft their $\frac{13 + 3\sqrt{7}}{3 + \sqrt{7}}$<br>their denominator must have 2 terms |
| | (numerator =)<br>$39 - 13\sqrt{7} + 9\sqrt{7} - 21$<br>or $18 - 4\sqrt{7}$ | M1dep | ft their numerator which must have 2 terms<br>oe<br>dep on 2nd M1 |
| | (denominator =) 2 | M1dep | ft their denominator<br>dep on 2nd M1 |
| | $9 - 2\sqrt{7}$ | A1 | |
| | **Alternative method 2** Works out $2 \times (13 + 3\sqrt{7})$ | | |
| | $\frac{1}{2} \times (6 + 2\sqrt{7}) \times AD$ | M1 | oe eg $(6 + 2\sqrt{7}) \times AD = (26 + 6\sqrt{7})$<br>may be implied |
| | $\frac{26 + 6\sqrt{7}}{6 + 2\sqrt{7}} \times \frac{6 - 2\sqrt{7}}{6 - 2\sqrt{7}}$ | M1 | ft their $\frac{26 + 6\sqrt{7}}{6 + 2\sqrt{7}}$<br>their denominator must have 2 term |
| | (numerator =)<br>$156 - 52\sqrt{7} + 36\sqrt{7} - 84$<br>or $72 - 16\sqrt{7}$ | M1dep | ft their numerator which must have 2 terms<br>oe<br>dep on 2nd M1 |
| | (denominator =) 8 | M1dep | ft their denominator<br>dep on 2nd M1 |
| | $9 - 2\sqrt{7}$ | A1 | |

**Mark scheme and Additional Guidance continue on the next page**

| | | | |
|---|---|---|---|
| **21 cont** | **Alternative method 3** Using identities | | |
| | $(6 + 2\sqrt{7}) \times AD = (26 + 6\sqrt{7})$ | M1 | oe |
| | $(6 + 2\sqrt{7})\times(a + b\sqrt{7})=(26 + 6\sqrt{7})$ | M1 | oe |
| | $6a + 14b = 26$ and $2a + 6b = 6$ | M1dep | oe eg $3a + 7b = 13$ and $a + 3b = 3$ |
| | $a = 9$ or $b = –2$ | M1dep | |
| | $9 – 2\sqrt{7}$ | A1 | |
| | **Additional Guidance** | | |
| | Alt1 $\frac{18 – 4\sqrt{7}}{2}$ | | M4 |
| | Alt2 $\frac{72 – 16\sqrt{7}}{8}$ | | M4 |
| | Omission of $\frac{1}{2}$ can score up to M0M1M1M1A0<br>eg $\frac{13 + 3\sqrt{7}}{6 + 2\sqrt{7}} \times \frac{6 – 2\sqrt{7}}{6 – 2\sqrt{7}}$<br>$\frac{78 – 26\sqrt{7} + 18\sqrt{7} – 42}{8}$<br>$4.5 – \sqrt{7}$<br>(If the $\frac{1}{2}$ is recovered then all 5 marks are possible) | | M0M1<br>M1M1<br>A0 |
| | Missing brackets must be recovered | | |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 22 | **Alternative method 1** Working with powers of 2 | | |
| | $2^{3x}$ or $2^{52}$ | B1 | from $8^x$ and $4^{26}$ |
| | $2^{52}(2^4 - 1)$ or $2^{52} \times 15$<br>or $2 \times 8^x = 2^{52}$ or $2 \times 2^{3x} = 2^{52}$ | M1 | oe |
| | $2^{3x+1} = 2^{52}$ or $3x + 1 = 52$ | M1dep | oe eg $2^{3x} = 2^{51}$ |
| | 17 | A1 | |
| | **Alternative method 2** Working with powers of 4 | | |
| | $4^{1.5x}$ or $4^{28}$ | B1 | from $8^x$ and $2^{56}$ |
| | $4^{26}(4^2 - 1)$ or $4^{26} \times 15$<br>or $2 \times 4^{1.5x} = 4^{26}$ | M1 | oe |
| | $4^{1.5x} = \frac{4^{26}}{4^{0.5}}$ or $1.5x = 26 - 0.5$ | M1dep | oe eg $4^{1.5x} = 4^{25.5}$ |
| | 17 | A1 | |
| | **Alternative method 3** Working with powers of 8 | | |
| | $8^{\frac{56}{3}}$ or $8^{\frac{52}{3}}$ | B1 | from $2^{56}$ and $4^{26}$ |
| | $8^{\frac{52}{3}}(8^{\frac{4}{3}} - 1)$ or $8^{\frac{52}{3}} \times 15$<br>or $2 \times 8^x = 8^{\frac{52}{3}}$ | M1 | oe |
| | $8^x = 8^{\frac{52}{3}} \div 8^{\frac{1}{3}}$ or $x = \frac{52}{3} - \frac{1}{3}$ | M1dep | oe eg $8^x = 8^{17}$ |
| | 17 | A1 | |
| | **Additional Guidance** | | |
| | | | |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 23(a) | Angles in the same segment | B1 | oe eg angles at the circumference are equal |
| | Alternate angles | B1 | do not accept alternative or alternating |
| | **Additional Guidance** | | |
| | Angles on the circumference from a chord | | B1 |
| | Angles in the same sector, opposite angles, parallel lines, angles from a chord, similar triangles, isosceles triangle, corresponding angles, triangles on a chord, intersecting chords, allied angles, alternate segment theorem | | B0 |

| Question | Answer | Mark | Comments |
|---|---|---|---|
| 23(b) | $\angle HJF = 3y$<br>or<br>$\angle JFG = 2x$<br>or<br>$\angle HFL = 2x$ | M1 | may be on the diagram<br>implied by one correct equation in $x$ and $y$ |
| | $2x + 3y + 98 = 180$<br>and<br>$4x + 7y = 180$ | M1dep | two correct equations in $x$ and $y$ |
| | A correct attempt to eliminate one of the variables from the two equations | M1dep | eg $(4x + 7y) – 2(2x + 3y)$ |
| | $x = 17$ and $y = 16$ | A1 | |
| | **Additional Guidance** | | |
| | | | |