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General Certificate of Education

Mathematics 6360

MPC2 Pure Core 2

Mark Scheme

2009 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC2

Q	Solution	Marks	Total	Comments
1(a)	{Area of sector =} $\frac{1}{2}r^2\theta$	M1		$\frac{1}{2}r^2\theta$ stated or used for area of sector. PI
	$=\frac{1}{2}\times 10^2 \times 0.8 = 40$ {cm^2}	A1	2	
(b)(i)	{Arc =} $r\theta$ …. = 8	M1 A1		$r\theta$ stated or used for arc length. PI PI
	Perimeter = 20 + $r\theta$ = 28 (cm)	A1ft	3	ft on 20 + $r\times\theta$
(ii)	Area of square = $\left[\frac{\text{c's answer for (b)(i)}}{4}\right]^2$	M1		PI
	= 49 {cm^2}	A1cao	2	
	Total		**7**	
2(a)	$h = 1.5$ $f(x) = x^2\sqrt{x^2-1}$ Integral = h/2 {….}	B1		PI
	{….} = $f(1.5)+2[f(3)+f(4.5)]+f(6)$	M1		For the M1 covered range must be 1.5 to 6 OE summing of areas of the three traps.
	{….} = 2.51(5..)+2[25.4(5..)+88.8(4..)]+212(.9..)	A1		Check at least 3sf values, rounded or truncated, or award if a combined value WRT 444 is seen or final answer is 333 or rounds to 333 Condone one numerical slip
	Integral = 0.75 × 444.1 = 333 to 3sf	A1cao	4	Must have 333 Treat using 4 strips as a MR and mark with max of B0M1A1A1cao as follows: h =1.125 B0 {….} =f(1.5)+2[f(2.625)+f(3.75)+f(4.875)]+f(6) M1 =2.51(5)+2[16.7(2)+50.8(2)+113(.3)]+212(.9) A1 or award if a combined value WRT 577 is seen or final answer is 325 or rounds to 325. Condone one numerical slip. Answer = 325 A1cao Must have 325
(b)	Increase the number of ordinates	E1	1	OE eg increase the number of strips
	Total		**5**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	{Area =} $\frac{1}{2}\times 7.4\times 5.26\times \sin 63°$	M1		
	= 17.3(407…) {m^2}	A1	2	Accept any value from 17.3 to 17.341
(b)	$\{BC^2 =\}\ 5.26^2 + 7.4^2 - 2\times 5.26\times 7.4\cos 63$	M1		RHS of cosine rule used
	….. = 27.66(76) + 54.76 − 35.34(22…)	m1		Correct order of evaluation
	$\Rightarrow BC = \sqrt{47.08(5...)} = 6.861(8..)$			
	BC = 6.86 {m} to 3sf	A1	3	AG. Cand. must show a 4th sf in either $\sqrt{47.08(5...)}$ or 6.861(8) before giving the printed answer 6.86
(c)	$\frac{\sin B}{5.26} = \frac{\sin 63}{BC}$	M1		Sine rule involving 'sin *B*' [If valid cosine rule used to find cos *B*, no marks awarded until stage of converting to sin *B*]
	sin *B* = 0.68 to 2sf	A1	2	If not 0.68, accept AWRT any value from 0.682 to 0.684 inclusive
	ALTn			
	$\frac{1}{2}\times 7.4\times (6.86..)\times \sin B = \text{c's ans (a)}$	(M1)		(6.86..) could be c's ans (b)
	sin *B* = 0.68 to 2sf	(A1)		If not 0.68, accept AWRT any value from 0.682 to 0.684 inclusive
	Total		**7**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$\frac{dy}{dx} = 3x^{\frac{1}{2}}$	M1		$kx^{\frac{1}{2}}$ with or without + c
	$= 6$ {when $x = 4$}	A1cao	2	Must be 6 and seen in (a)(i) $6 + c$ is A0
(ii)	y-coordinate of A = $2\times 4^{\frac{3}{2}}$ (= 16)	M1		Substitute $x = 4$ in $y = 2x^{\frac{3}{2}}$
	$6 \times m' = -1$	M1		$m_1 \times m_2 = -1$ OE used with c's value of $\frac{dy}{dx}$ when $x = 4$. PI
	$y - 16 = m(x - 4)$	m1		dep on 1st M1 in (a)(ii) m must be numerical
	$y - 16 = -\frac{1}{6}(x-4)$	A1	4	ACF
(b)(i)	$\int 8x^{\frac{1}{2}}\,dx = \frac{8}{3/2}x^{\frac{1}{2}+1}$ {+ c}	M1		Index raised by 1
	$= \frac{16}{3}x^{\frac{3}{2}}$ {+ c}	A1	2	Condone missing '+ c' Coefficient must be simplified
(ii)	$\int 2x^{\frac{3}{2}}\,dx = \frac{2}{5/2}x^{\frac{5}{2}}$ {+ c} {$= \frac{4}{5}x^{\frac{5}{2}}$ {+ c}}	B1		Can award for unsimplified form
	$\int_0^4 8x^{\frac{1}{2}}\,dx - \int_0^4 2x^{\frac{3}{2}}\,dx$	M1		Ignore limits here
	$= \frac{16}{3}(4)^{\frac{3}{2}} - 0 - \left[\frac{4}{5}(4)^{\frac{5}{2}} - 0\right]$	M1		F(4) − F(0) used in either; {F(0)=0 PI} Cand. must be using F(x) as a result of his/her integration in (b)(i) or in the (b)(ii) B1 line above
	$= \frac{256}{15}$	A1	4	Accept any value from 17.04 to 17.1 inclusive in place of 256/15
(c)	Translation	B1		Accept 'translat…' as equivalent [T or Tr is NOT sufficient]
	$\begin{bmatrix} -3 \\ 0 \end{bmatrix}$	B1	2	Accept equivalent in words provided linked to 'translation/move/shift' (B0B0 if >1 transformation)
	Total		**14**	

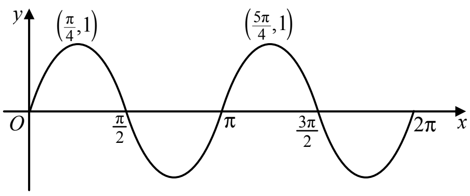
MPC2 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$(1+2x)^4 = 1+4(2x)+6(2x)^2+4(2x)^3+(2x)^4$	M1		(1), 4, 6, 4, (1) OE unsimplified with correct powers of x Algebraic multiplication must be a full method
	$= 1 + 8x$	A1		Accept $a = 8$ provided 1st term is 1
	$+ 24x^2$	A1		$b = 24$
	$+ 32x^3$ $\{+ 16x^4\}$	A1	4	$c = 32$
(b)	$(1 - 2x)^4 = 1 - 8x + 24x^2 - 32x^3$ $\{+ 16x^4\}$	M1 A1ft		Replace x by $-x$ even in M1 line of (a) PI ft c's non zero values for a, b and c
	$(1 + 2x)^4 + (1 - 2x)^4$ $= 1 + 8x + 24x^2 + 32x^3 + 16x^4$ $+ 1 - 8x + 24x^2 - 32x^3 + 16x^4$ $= 2 + 48x^2 + 32x^4$	A1cso	3	AG Be convinced
(c)	$\frac{dy}{dx} = 96x + 128x^3$	M1		A correct power of x OE
	For st. pt. $96x + 128x^3 = 0$	A1		
	$32x(3 + 4x^2) = 0$ Since $3+4x^2 > 0$ there is only one stationary point	E1		Any valid explanation of curve having just one stationary point
	The coordinates of the stationary point are (0, 2)	B1	4	(0, 2) as the **only** stationary point
	Total		**11**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\log_a 40$	B1	1	Accept '$k = 40$'
(ii)	$\log_a 8$	B1	1	Accept '$k = 8$'
(iii)	$\log_a 125$	B1	1	Accept '$k = 125$' but not '$k = 5^3$'
(b)	$\log_{10}\left[(1.5)^{3x}\right] = \log_{10} 7.5$	M1		Correct statement having taken logs of both sides of $(1.5)^{3x} = 7.5$ OE PI or $3x = \log_{1.5} 7.5$ seen
	$3x\log_{10} 1.5 = \log_{10} 7.5$	m1		$\log 1.5^{3x} = 3x\log 1.5$ OE
	$x = \dfrac{\lg 7.5}{3\lg 1.5} = 1.65645....$ = 1.656 to 3dp	A1	3	Both method marks must have been awarded with clear use of logarithms seen
(c)	$\log_2 p = m \Rightarrow p = 2^m$; $\log_8 q = n \Rightarrow q = 8^n$	M1		Either $p = 2^m$ or $q = 8^n$ seen or used
	$p = 2^m$ and $q = 2^{3n}$	m1		Writing $8^n = 2^{3n}$ and having $p = 2^m$
	$pq = 2^m \times \left(2^3\right)^n = 2^m \times 2^{3n}$ so $pq = 2^{m+3n}$	A1	3	Accept $y = m + 3n$
	Total		**9**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\{x =\}$ $\sin^{-1}(0.8) = 0.927(29\ldots)$ $\{=\beta\}$	M1		$\sin^{-1}(0.8)$ PI
	$\{x =\}$ $\pi - \beta$	m1		
	$x =$ 0.927(29…), 2.21(42…)	A1	3	Both Ignore values outside interval 0–2π but A0 if 'extra' values inside the given interval
(b)(i)	$\left(\frac{3\pi}{2}, -1\right)$	B2,1	2	B1 if one coordinate correct or $\left(-1, \frac{3\pi}{2}\right)$
(ii)	$\pi - \alpha$	B1	1	
(iii)	$RS = (2\pi - \alpha) - (\pi + \alpha)$	M1		OE eg $RS = PQ = (\pi - \alpha) - \alpha$
	$= \pi - 2\alpha$	A1	2	Must be simplified
(c)	 	B1		Sine curve with positive gradient at *O* with at least 3 stationary points between 0 and 2π
		B1		Correct shaped curve with 2 max and 2 min between 0 and 2π
		B1		All 5 correct points of intersection with *x*-axis with $\frac{\pi}{2}$, π and $\frac{3\pi}{2}$ clearly shown
	Maximum points $\left(\frac{\pi}{4}, 1\right)$ and $\left(\frac{5\pi}{4}, 1\right)$ stated or clearly shown on the sketch	B2,1	5	B1 for either: 1 as the *y*-coordinate of max pt(s) or: two max pts between 0 and 2π with correct *x*-coordinates
	Total		**13**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
8(a)	{S_{40} =} $\frac{40}{2}\left[2a+(40-1)d\right]$	M1		
	20(2a + 39d) = 1250	A1		
	{25th term =} $a + (25 - 1)d$	M1		
	$a + 24d = 38$	A1		
		m1		Dep on both previous two Ms. Solving two equations in a and d simultaneously
	$18d = 27 \Rightarrow d = 1.5$	A1cso	6	AG Be convinced SC Using the given answer for d: mark out of a maximum of 4/6 as M1A1M1A1{conclusion also needed in last A mark} (m0A0)
(b)	$a = 38 - 24 \times 1.5$	M1		PI if using $a = 2$ **in (b)**
	$= 2$			If using eg $a = 38$ award this M mark at stage: no. of terms $\frac{100-38}{1.5}+1+24$
	$a + (n - 1)1.5 < 100$	M1		
	$n < \frac{100-a}{1.5}+1$			
	$n < 66.333$….. $\Rightarrow$ number of terms < 100 is 66	A1	3	NMS mark as B3 for 66 else B0
	Total		**9**	
	TOTAL		**75**	