AQA

Please write clearly in block capitals.

Centre number | | | | | Candidate number | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

A-level
MATHEMATICS

Unit Pure Core 4

Friday 16 June 2017   Afternoon   Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Answer **all** questions.

Answer each question in the space provided for that question.

1 A curve is defined by the parametric equations

$$x = (t-1)^3 \quad , \quad y = 3t - \frac{8}{t^2} \qquad t \neq 0$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

[3 marks]

(b) Find the equation of the normal at the point on the curve where $t = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 1**

QUESTION PART REFERENCE	Answer space for question 1

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2 (a) Express $7\cos x + 3\sin x$ in the form $R\cos(x-\alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving your value of α to the nearest $0.1°$.

[3 marks]

(b) Use your answer to part **(a)** to solve the equation $7\cos 2\theta + 3\sin 2\theta = 5$ in the interval $0° < \theta < 180°$, giving your solutions to the nearest $0.1°$.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 2**

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QUESTION PART REFERENCE	Answer space for question 2

3 (a) The polynomial $f(x)$ is defined by $f(x) = 6x^3 - 11x^2 + 2x + 8$.

(i) Use the Factor Theorem to show that $(3x + 2)$ is a factor of $f(x)$.

[2 marks]

(ii) Show that $f(x)$ has no other linear factors.

[4 marks]

(b) The polynomial $g(x)$ is defined by $g(x) = f(x) - (6x^2 - 2x - 4)$.

Given that $(3x + 2)$ is a factor of $g(x)$, express $g(x)$ as a product of three linear factors.

[2 marks]

(c) The function h is defined by $h(x) = \dfrac{g(x)}{6x^3 - 5x^2 - 6x}$.

Show that $h(x)$ can be simplified to the form $p + qx^n$ where p, q and n are integers.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 3**

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QUESTION PART REFERENCE

Answer space for question 3

Turn over ►

4 (a) Find the binomial expansion of $(1-4x)^{-\frac{1}{2}}$ up to and including the term in x^2.

[2 marks]

(b) Find the binomial expansion of $(16+4x)^{\frac{3}{4}}$ up to and including the term in x^2.

[3 marks]

(c) Hence find the expansion of $\sqrt{\dfrac{(16+4x)^{\frac{3}{2}}}{(1-4x)}}$ in ascending powers of x up to and including the term in x^2.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 4**

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QUESTION PART REFERENCE	**Answer space for question 4**

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5 (a) By replacing 3θ by $(2\theta+\theta)$ show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

[4 marks]

(b) By using the result from part **(a)** and assuming that $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, find the exact value of

$$\int_0^{\frac{\pi}{6}} \left(2\sin^3\theta + 3\right)\, \mathrm{d}\theta$$

[6 marks]

QUESTION PART REFERENCE	Answer space for question 5

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QUESTION PART REFERENCE	**Answer space for question 5**

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6 The line l_1 has equation $\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ -2 \\ p \end{bmatrix}$ where p is an integer.

The line l_2 has equation $\mathbf{r} = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} + \mu \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$.

The points A and C have coordinates $(3, 1, -1)$ and $(2, 1, -3)$ respectively.

(a) The point A lies on l_1. Show that $p = 4$.

[2 marks]

(b) Show that the lines l_1 and l_2 are perpendicular.

[1 mark]

(c) Show that the lines l_1 and l_2 do **not** intersect.

[3 marks]

(d) The point B lies on l_1 such the triangle ABC is isosceles with $AC = BC$.
Find the coordinates of B.

[6 marks]

QUESTION PART REFERENCE	**Answer space for question 6**

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7 A curve C is defined by the equation

$$\sin 3y + 3\mathrm{e}^{-2x}y + 2x^2 = 5$$

(a) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

[6 marks]

(b) (i) Show that, at the points on C where $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$, $y = rx\mathrm{e}^{2x}$, where r is a rational number.

[2 marks]

(ii) Hence show that there is a point on C in the interval $1 < x < 1.2$ where $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$.

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 7**

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QUESTION PART REFERENCE	Answer space for question 7

8 (a) It is given that $\dfrac{1}{x(k-x)}$ can be expressed as $A\left(\dfrac{1}{x}+\dfrac{1}{k-x}\right)$ where A and k are positive constants. Find A in terms of k.

[2 marks]

(b) A rumour is spreading through a school of 1200 pupils. The rate at which the rumour is spreading can be modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{x(1200-x)}{3600}$$

where x is the number of pupils who have heard the rumour t hours after 11.00 am.

By 11.00 am, 300 pupils have heard the rumour. Taking $t=0$ as 11.00 am, use integration to solve this differential equation to show that

$$t=3\ln\left(\frac{3x}{1200-x}\right)$$

[5 marks]

(c) Use this model to:

(i) find the time of day by which half of the pupils will have heard the rumour, giving your answer to the nearest 5 minutes

[2 marks]

(ii) find x in terms of t and hence find the number of pupils who will have heard the rumour by 3.00 pm.

[3 marks]

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END OF QUESTIONS