

## A Level Further Mathematics B (MEI)

### Y421 Mechanics Major

#### Sample Question Paper

Version 2

## Date – Morning/Afternoon

Time allowed: 2 hours 15 minutes

#### You must have:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

#### You may use:

- a scientific or graphical calculator



### INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

### INFORMATION

- The total number of marks for this paper is **120**.
- The marks for each question are shown in brackets [ ].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

## Section A (26 marks)

Answer **all** the questions

- 1 A particle P has position vector  $\mathbf{r}$  m at time  $t$  s given by  $\mathbf{r} = (t^3 - 3t^2)\mathbf{i} - (4t^2 + 1)\mathbf{j}$  for  $t \geq 0$ .  
Find the magnitude of the acceleration of P when  $t = 2$ . [4]
- 2 A particle of mass 5 kg is moving with velocity  $2\mathbf{i} + 5\mathbf{j}$  m s<sup>-1</sup>. It receives an impulse of magnitude 15 N s in the direction  $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ . Find the velocity of the particle immediately afterwards. [3]
- 3 The fixed points E and F are on the same horizontal level with  $EF = 1.6$  m. A light string has natural length 0.7 m and modulus of elasticity 29.4 N. One end of the string is attached to E and the other end is attached to a particle of mass  $M$  kg. A second string, identical to the first, has one end attached to F and the other end attached to the particle. The system is in equilibrium in a vertical plane with each string stretched to a length of 1 m, as shown in Fig. 3.

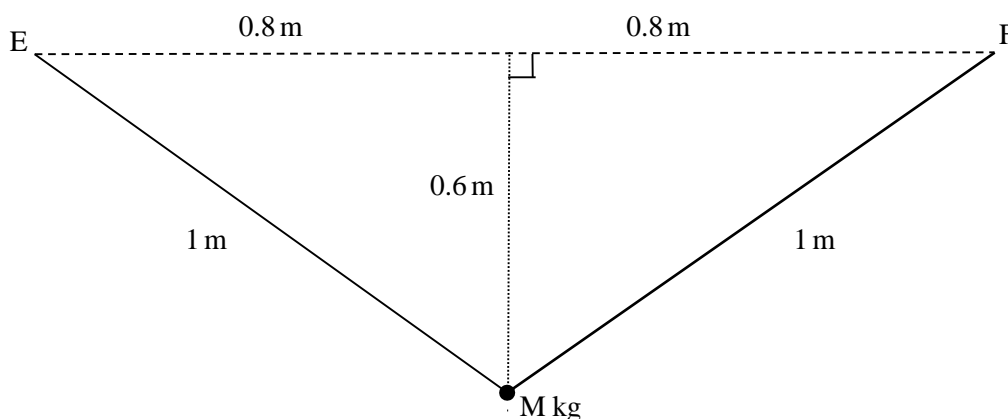


Fig. 3

- (i) Find the tension in each string. [2]
- (ii) Find  $M$ . [3]

- 4 A fixed smooth sphere has centre O and radius a. A particle P of mass m is placed at the highest point of the sphere and given an initial horizontal speed u.

For the first part of its motion, P remains in contact with the sphere and has speed v when OP makes an angle  $\theta$  with the upward vertical. This is shown in Fig. 4.

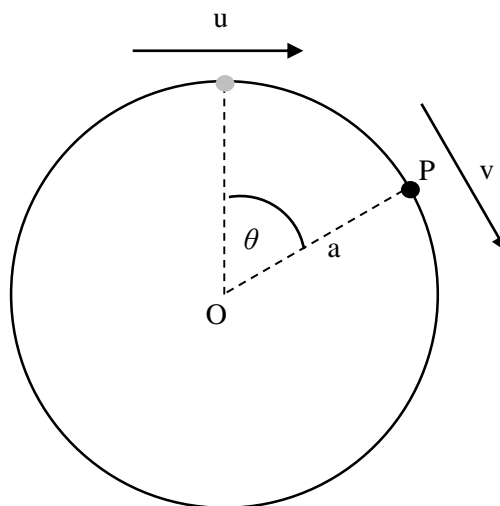


Fig. 4

- (i) By considering the energy of P, show that  $v^2 = u^2 + 2ga(1 - \cos \theta)$ . [2]

- (ii) Show that the magnitude of the normal contact force between the sphere and particle P is  $mg(3\cos \theta - 2) - \frac{mu^2}{a}$ . [2]

The particle loses contact with the sphere when  $\cos \theta = \frac{3}{4}$ .

- (iii) Find an expression for u in terms of a and g. [2]

- 5 Fig. 5 shows a light inextensible string of length 3.3 m passing through a small smooth ring R. The ends of the string are attached to fixed points A and B, where A is vertically above B. The ring R has mass 0.27 kg and is moving with constant speed in a horizontal circle of radius 1.2 m. The distances AR and BR are 2 m and 1.3 m respectively.

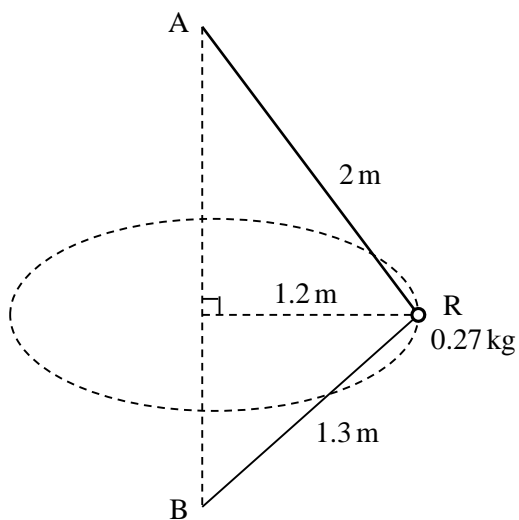


Fig. 5

- (i) Show that the tension in the string is 6.37 N. [4]
- (ii) Find the speed of R. [4]

## Section B (94 marks)

Answer **all** the questions

- 6 Fig. 6 shows a pendulum which consists of a rod AB freely hinged at the end A with a weight at the end B. The pendulum is oscillating in a vertical plane. The total energy,  $E$ , of the pendulum is given by

$$E = \frac{1}{2} I \omega^2 - mgh \cos \theta ,$$

where

- $\omega$  is its angular speed
- $m$  is its mass
- $h$  is the distance of its centre of mass from A
- $\theta$  is the angle the rod makes with the downward vertical
- $g$  is the acceleration due to gravity
- $I$  is a quantity known as the moment of inertia of the pendulum.

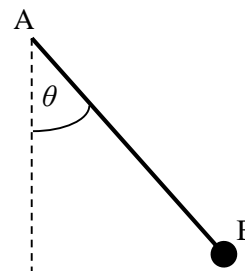


Fig. 6

- (i) Use the expression for  $E$  to deduce the dimensions of  $I$ . [4]

It is suggested that the period of oscillation,  $T$ , of the pendulum is given by  $T = kI^\alpha (mg)^\beta h^\gamma$ , where  $k$  is a dimensionless constant.

- (ii) Use dimensional analysis to find the values of  $\alpha$ ,  $\beta$  and  $\gamma$ . [5]

A class experiment finds that, when all other quantities are fixed,  $T$  is proportional to  $\frac{1}{\sqrt{m}}$ .

- (iii) Determine whether this result is consistent with your answer to part (ii). [1]

- 7 A uniform ladder of length 8 m and weight 180 N stands on a rough horizontal surface and rests against a smooth vertical wall. The ladder makes an angle of  $20^\circ$  with the wall. A woman of weight 720 N stands on the ladder. Fig. 7 shows this situation modelled with the woman's weight acting at a distance  $x$  m from the lower end of the ladder.

The system is in equilibrium.

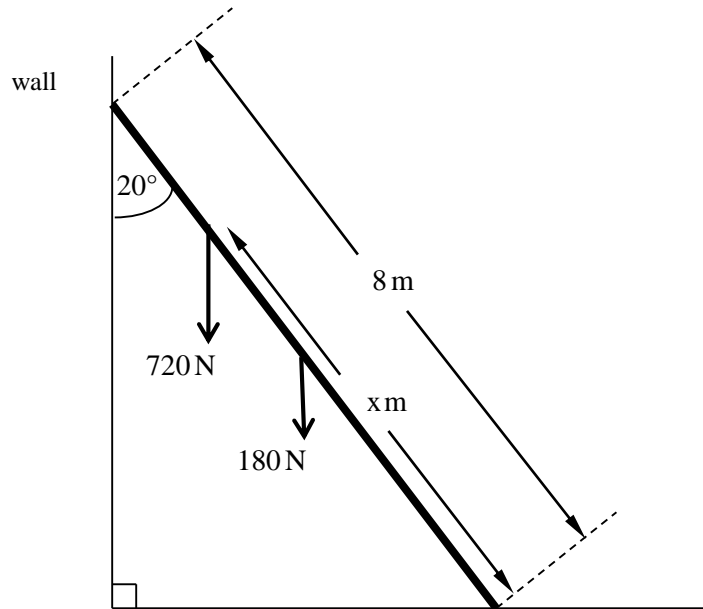


Fig. 7

- (i) Show that the frictional force between the ladder and the horizontal surface is  $F$  N, where  $F = 90(1 + x) \tan 20^\circ$ . [4]
- (ii) (A) State with a reason whether  $F$  increases, stays constant or decreases as  $x$  increases. [1]
- (B) Hence determine the set of values of the coefficient of friction between the ladder and the surface for which the woman can stand anywhere on the ladder without it slipping. [4]

- 8** A tractor has a mass of 6000 kg. When developing a power of 5 kW, the tractor is travelling at a steady speed of  $2.5 \text{ m s}^{-1}$  across a horizontal field.

(i) Calculate the magnitude of the resistance to the motion of the tractor. [2]

The tractor comes to horizontal ground where the resistance to motion is different. The power developed by the tractor during the next 10 s has an average value of 8 kW. During this time, the tractor accelerates uniformly from  $2.5 \text{ m s}^{-1}$  to  $3 \text{ m s}^{-1}$ .

(ii) (A) Show that the work done against the resistance to motion during the 10 s is 71 750 J. [4]

(B) Assuming that the resistance to motion is constant, calculate its value. [3]

The tractor can usually travel up a straight track inclined at an angle  $\alpha$  to the horizontal, where  $\sin \alpha = \frac{1}{20}$ , while accelerating uniformly from  $3 \text{ m s}^{-1}$  to  $3.25 \text{ m s}^{-1}$  over a distance of 100 m against a resistance to motion of constant magnitude of 2000 N.

The tractor develops a fault which limits its maximum power to 16 kW.

(iii) Determine whether the tractor could now perform the same motion up the track.  
[You should assume that the mass of the tractor and the resistance to motion remain the same.] [7]

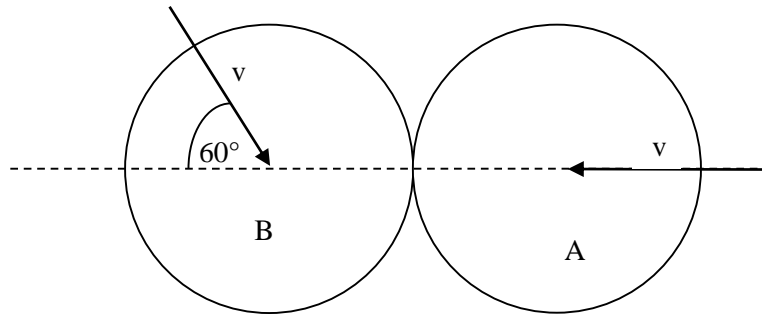


Fig. 9

Fig. 9 shows the instant of impact of two identical uniform smooth spheres, A and B, each with mass  $m$ . Immediately before they collide, the spheres are sliding towards each other on a smooth horizontal table in the directions shown in the diagram, each with speed  $v$ . The coefficient of restitution between the spheres is  $\frac{1}{2}$ .

- (i) Show that, immediately after the collision, the speed of A is  $\frac{1}{8}v$ . Find its direction of motion. [6]
- (ii) Find the percentage of the original kinetic energy that is lost in the collision. [7]
- (iii) State where in your answer to part (i) you have used the assumption that the contact between the spheres is smooth. [1]



**10 In this question take  $g = 10$ .**

A smooth ball of mass  $0.1 \text{ kg}$  is projected from a point on smooth horizontal ground with speed  $65 \text{ m s}^{-1}$  at an angle  $\alpha$  to the horizontal, where  $\tan \alpha = \frac{3}{4}$ . While it is in the air the ball is modelled as a particle moving freely under gravity. The ball bounces on the ground repeatedly. The coefficient of restitution for the first bounce is  $0.4$ .

- (i) Show that the ball leaves the ground after the first bounce with a horizontal speed of  $52 \text{ m s}^{-1}$  and a vertical speed of  $15.6 \text{ m s}^{-1}$ . Explain your reasoning carefully. [4]
- (ii) Calculate the magnitude of the impulse exerted on the ball by the ground at the first bounce. [2]

Each subsequent bounce is modelled by assuming that the coefficient of restitution is  $0.4$  and that the bounce takes no time. The ball is in the air for  $T_1$  seconds between projection and bouncing the first time,  $T_2$  seconds between the first and second bounces, and  $T_n$  seconds between the  $(n-1)$ th and  $n$ th bounces.

- (iii) (A) Show that  $T_1 = \frac{39}{5}$ . [2]
- (B) Find an expression for  $T_n$  in terms of  $n$ . [2]
- (iv) According to the model, how far does the ball travel horizontally while it is still bouncing? [3]
- (v) According to the model, what is the motion of the ball after it has stopped bouncing? [1]

- 11 The region bounded by the x-axis and the curve  $y = \frac{1}{2}k(1-x^2)$  for  $-1 \leq x \leq 1$  is occupied by a uniform lamina, as shown in Fig. 11.1.

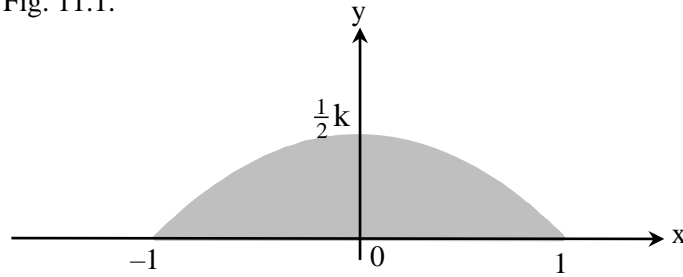


Fig. 11.1

- (i) In this question you must show detailed reasoning.

Show that the centre of mass of the lamina is at  $\left(0, \frac{1}{5}k\right)$ .

[7]

A shop sign is modelled as a uniform lamina in the form of the lamina in part (i) attached to a rectangle ABCD, where  $AB = 2$  and  $BC = 1$ . The sign is suspended by two vertical wires attached at A and D, as shown in Fig. 11.2.

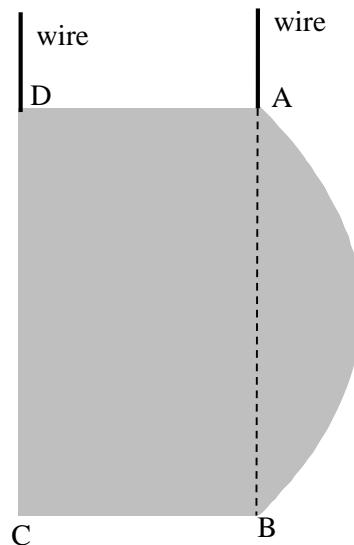


Fig. 11.2

- (ii) Show that the centre of mass of the sign is at a distance

$$\frac{2k^2 + 10k + 15}{10k + 30}$$

from the midpoint of CD.

[4]

The tension in the wire at A is twice the tension in the wire at D.

- (iii) Find the value of  $k$ .

[5]

- 12 Fig. 12 shows  $x$ - and  $y$ - coordinate axes with origin  $O$  and the trajectory of a particle projected from  $O$  with speed  $28 \text{ m s}^{-1}$  at an angle  $\alpha$  to the horizontal. After  $t$  seconds, the particle has horizontal and vertical displacements  $x \text{ m}$  and  $y \text{ m}$ .

Air resistance should be neglected.

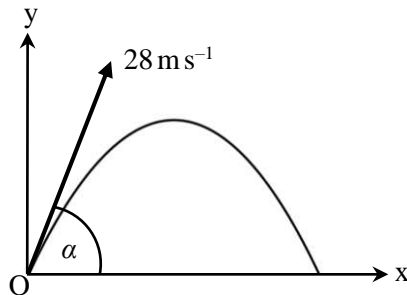


Fig. 12

- (i) Show that the equation of the trajectory is given by

$$\tan^2 \alpha - \frac{160}{x} \tan \alpha + \frac{160y}{x^2} + 1 = 0. \quad (*) \quad [5]$$

- (ii) (A) Show that if (\*) is treated as an equation with  $\tan \alpha$  as a variable and with  $x$  and  $y$  as constants, then (\*) has two distinct real roots for  $\tan \alpha$  when  $y < 40 - \frac{x^2}{160}$ . [3]

- (B) Show the inequality in part (ii) (A) as a locus on the graph of  $y = 40 - \frac{x^2}{160}$  in the Printed Answer Booklet and label it R. [1]

S is the locus of points  $(x, y)$  where (\*) has **one** real root for  $\tan \alpha$ .

T is the locus of points  $(x, y)$  where (\*) has **no** real roots for  $\tan \alpha$ .

- (iii) Indicate S and T on the graph in the Printed Answer Booklet. [2]

- (iv) State the significance of R, S and T for the possible trajectories of the particle. [3]

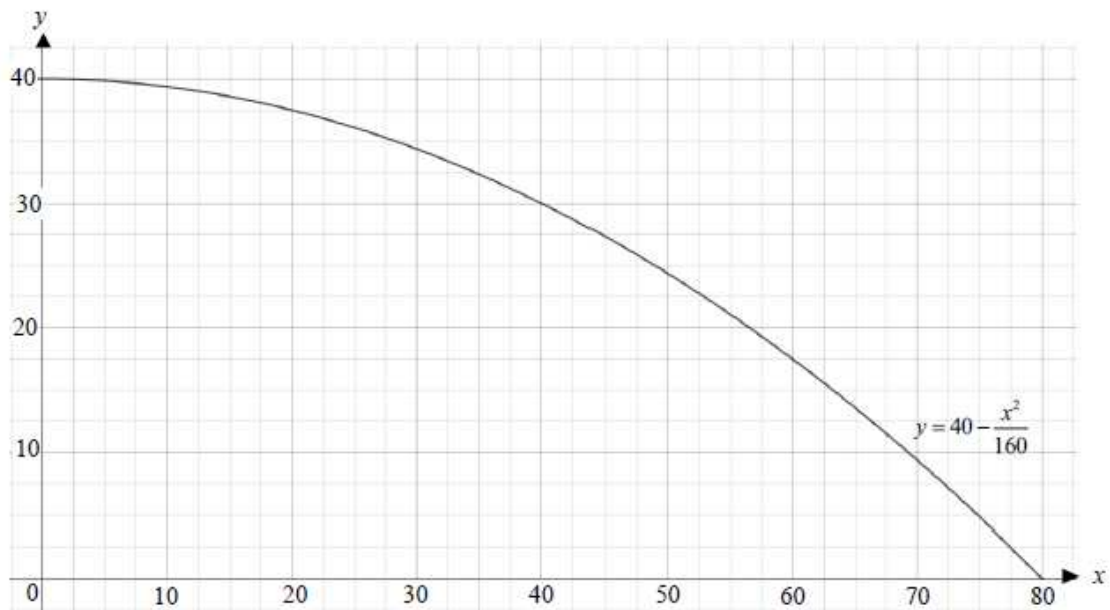
A machine can fire a tennis ball from ground level with a maximum speed of  $28 \text{ m s}^{-1}$ .

- (v) State, with a reason, whether a tennis ball fired from the machine can achieve a range of 80 m. [1]

**END OF QUESTION PAPER**

12 (v)

Spare copy of graph for question 12 (ii) and 12 (iii)

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