4754 (C4) Applications of Advanced Mathematics

Section A

1 $\frac{x}{x^2-4}+\frac{2}{x+2}=\frac{x}{(x-2)(x+2)}+\frac{2}{x+2}$ $=\frac{x+2(x-2)}{(x+2)(x-2)}$ $=\frac{3x-4}{(x+2)(x-2)}$	M1 M1 A1 [3]	combining fractions correctly factorising and cancelling (may be $3x^2+2x-8$)
2 $V=\int_0^1 \pi y^2 dx = \int_0^1 \pi(1+e^{2x})dx$ $=\pi\left[x+\frac{1}{2}e^{2x}\right]_0^1$ $=\pi(1+\frac{1}{2}e^2-\frac{1}{2})$ $=\frac{1}{2}\pi(1+e^2)$ *	M1 B1 M1 E1 [4]	must be π x their y^2 in terms of x $\left[x+\frac{1}{2}e^{2x}\right]$ only substituting both x limits in a function of x www
3 cos 2θ = sin θ ⇒ 1 − 2sin²θ = sin θ ⇒ 1 − sin θ − 2sin²θ = 0 ⇒ (1 − 2sin θ)(1 + sin θ) = 0 ⇒ sin θ = ½ or −1 ⇒ θ = π/6, 5π/6, 3π/2	M1 M1 A1 M1 A1 A2,1,0 [7]	cos2θ = 1 − 2sin²θ oe substituted forming quadratic(in one variable) =0 correct quadratic www factorising or solving quadratic ½ , −1 oe www cao penalise extra solutions in the range
4 sec θ = x/2 , tan θ = y/3 sec²θ = 1 + tan²θ ⇒ $x^2/4 = 1 + y^2/9$ ⇒ $x^2/4 - y^2/9 = 1$ * **OR** $x^2/4-y^2/9= 4\sec^2\theta/4-9\tan^2\theta/9$ $=\sec^2\theta-\tan^2\theta =1$	M1 M1 E1 [3]	sec²θ = 1 + tan²θ used (oe, e.g. converting to sines and cosines and using cos²θ + sin²θ = 1) eliminating θ (or x and y) www
5(i) dx/du = 2u , dy/du = 6u^2 ⇒ $\frac{dy}{dx}=\frac{dy/du}{dx/du}=\frac{6u^2}{2u}$ $= 3u$ **OR** $y=2(x-1)^{3/2}$, $dy/dx=3(x-1)^{1/2}=3u$	B1 M1 A1 [3]	both 2u and 6u^2 B1(y=f(x)), M1 differentiation, A1
(ii) At (5, 16), u = 2 ⇒ dy/dx = 6	M1 A1 [2]	cao

6(i) $(1+4x^2)^{-\frac{1}{2}} = 1-\frac{1}{2}.4x^2+\frac{(-\frac{1}{2}).(-\frac{3}{2})}{2!}(4x^2)^2+...$ $= 1-2x^2+6x^4+...$ Valid for $-1 < 4x^2 < 1 \Rightarrow -½ < x < ½$	M1 A1 A1 M1A1 [5]	binomial expansion with p= -1/2 $1 - 2x^2$ … $+ 6x^4$
(ii) $\frac{1-x^2}{\sqrt{1+4x^2}} = (1-x^2)(1-2x^2+6x^4+...)$ $= 1-2x^2+6x^4-x^2+2x^4+...$ $= 1 - 3x^2 + 8x^4 + …$	M1 A1 A1 [3]	substitituting their $1-2x^2+6x^4+...$ and expanding ft their expansion (of three terms) cao
7 $\sqrt{3}\sin x - \cos x = R\sin(x-\alpha)$ $= R(\sin x\cos\alpha - \cos x\sin\alpha)$ $\Rightarrow$ $\sqrt{3} = R\cos\alpha$, $1 = R\sin\alpha$ $\Rightarrow$ $R^2 = 3 + 1 = 4 \Rightarrow R = 2$ $\tan\alpha = 1/\sqrt{3}$ $\Rightarrow \alpha = \pi/6$ $\Rightarrow$ $y = 2\sin(x - \pi/6)$ Max when $x - \pi/6 = \pi/2 \Rightarrow x = \pi/6 + \pi/2 = 2\pi/3$ max value $y = 2$ So maximum is $(2\pi/3, 2)$	 M1 B1 M1 A1 B1 B1 [6]	 correct pairs soi $R = 2$ ft cao www cao ft their R SC B1 $(2, 2\pi/3)$ no working

Section B

8(i) At A: 3×0 + 2×0 + 20×(–15) + 300 = 0 At B: 3×100 + 2×0 + 20×(–30) + 300 = 0 At C: 3×0 + 2×100 + 20×(–25) + 300 = 0 So ABC has equation 3*x* + 2*y* + 20 *z* + 300 = 0	M1 A2,1,0 [3]	substituting co-ords into equation of plane… for ABC **OR** using two vectors in the plane form vector product M1A1 then 3*x* +2*y* +20z = *c* = -300 A1 **OR** using vector equation of plane M1,elim both parameters M1, A1
(ii) $\overrightarrow{DE} = \begin{pmatrix}100\\0\\-10\end{pmatrix}$ $\overrightarrow{DF} = \begin{pmatrix}0\\100\\5\end{pmatrix}$	B1B1	
$\begin{pmatrix}100\\0\\-10\end{pmatrix}.\begin{pmatrix}2\\-1\\20\end{pmatrix} = 100\times2+0\times-1+-10\times20 = 200-200 = 0$	B1	need evaluation
$\begin{pmatrix}0\\100\\5\end{pmatrix}.\begin{pmatrix}2\\-1\\20\end{pmatrix} = 0\times2+100\times-1+5\times20 = -100+100 = 0$	B1	need evaluation
Equation of plane is 2*x* – *y* + 20*z* = *c* At D (say) *c* = 20×–40 = –800 So equation is 2*x* – *y* + 20*z* + 800 = 0	M1 A1 [6]	
(iii) Angle is θ, where $\cos\theta = \dfrac{\begin{pmatrix}2\\-1\\20\end{pmatrix}.\begin{pmatrix}3\\2\\20\end{pmatrix}}{\sqrt{2^2+(-1)^2+20^2}\sqrt{3^2+2^2+20^2}} = \dfrac{404}{\sqrt{405}\sqrt{413}}$ ⇒ θ = 8.95°	M1 A1 A1 A1cao [4]	formula with correct vectors top bottom (or 0.156 radians)
(iv) RS: $\mathbf{r} = \begin{pmatrix}15\\34\\0\end{pmatrix} + \lambda\begin{pmatrix}3\\2\\20\end{pmatrix}$	B1	$\begin{pmatrix}15\\34\\0\end{pmatrix}+\ldots$
$= \begin{pmatrix}15+3\lambda\\34+2\lambda\\20\lambda\end{pmatrix}$	B1	$\ldots+\lambda\begin{pmatrix}3\\2\\20\end{pmatrix}$
⇒ 3(15+3λ) + 2(34+2λ) +20.20λ+300 = 0 ⇒ 45 + 9λ + 68 + 4λ + 400 λ + 300 = 0	M1	solving with plane
⇒ 413 + 413λ = 0 ⇒ λ = –1	A1	λ = –1
so S is (12, 32, –20)	A1 [5]	cao

9(i) $v = \int 10e^{-\frac{1}{2}t}dt$ $= -20e^{-\frac{1}{2}t} + c$ when $t = 0$, $v = 0$ $\Rightarrow$ $0 = -20 + c$ $\Rightarrow$ $c = 20$ so $v = 20 - 20e^{-\frac{1}{2}t}$	M1 A1 M1 A1 [4]	separate variables and intend to integrate $-20e^{-\frac{1}{2}t}$ finding c cao
(ii) As $t \to \infty$ $e^{-1/2\,t} \to 0$ $\Rightarrow$ $v \to 20$ So long term speed is 20 m s^{-1}	M1 A1 [2]	ft (for their $c>0$, found)
(iii) $\frac{1}{(w-4)(w+5)} = \frac{A}{w-4} + \frac{B}{w+5}$ $= \frac{A(w+5)+B(w-4)}{(w-4)(w+5)}$ $\Rightarrow$ $1 \equiv A(w+5) + B(w-4)$ $w = 4$: $1 = 9A$ $\Rightarrow A = 1/9$ $w = -5$: $1 = -9B \Rightarrow B = -1/9$ $\Rightarrow \frac{1}{(w-4)(w+5)} = \frac{1/9}{w-4} - \frac{1/9}{w+5}$ $= \frac{1}{9(w-4)} - \frac{1}{9(w+5)}$	M1 M1 A1 A1 [4]	cover up, substitution or equating coeffs 1/9 −1/9
(iv) $\frac{dw}{dt} = -\frac{1}{2}(w-4)(w+5)$ $\Rightarrow \int \frac{dw}{(w-4)(w+5)} = \int -\frac{1}{2}dt$ $\Rightarrow \int [\frac{1}{9(w-4)} - \frac{1}{9(w+5)}]dw = \int -\frac{1}{2}dt$ $\Rightarrow \frac{1}{9}\ln(w-4) - \frac{1}{9}\ln(w+5) = -\frac{1}{2}t + c$ $\Rightarrow \frac{1}{9}\ln\frac{w-4}{w+5} = -\frac{1}{2}t + c$ When $t = 0$, $w = 10 \Rightarrow c = \frac{1}{9}\ln\frac{6}{15} = \frac{1}{9}\ln\frac{2}{5}$ $\Rightarrow \ln\frac{w-4}{w+5} = -\frac{9}{2}t + \ln\frac{2}{5}$ $\Rightarrow \frac{w-4}{w+5} = e^{-\frac{9}{2}t+\ln\frac{2}{5}} = \frac{2}{5}e^{-\frac{9}{2}t} = 0.4e^{-4.5t}$ *	M1 M1 A1ft M1 M1 E1 [6]	separating variables substituting their partial fractions integrating correctly (condone absence of c) correctly evaluating c (at any stage) combining lns (at any stage) www
(v) As $t \to \infty$ $e^{-4.5\,t} \to 0$ $\Rightarrow$ $w - 4 \to 0$ So long term speed is 4 m s^{-1}.	M1 A1 [2]	

Comprehension

1. (i)

2	1	3
3	**2**	**1**
1	**3**	**2**

B1 cao

(ii)

2	3	**1**
3	**1**	**2**
1	2	**3**

B1 cao

2. Dividing the grid up into four 2 x 2 blocks gives

1	2	3	4
3	1	4	2
2	4	1	3
4	3	2	1

Lines drawn on diagram or reference to 2 x 2 blocks. **M1**

One (or more) block does not contain all 4 of the symbols 1, 2, 3 and 4. oe. **E1**

3.

1	2	3	4
4	**3**	**1**	**2**
2	**1**	**4**	**3**
3	**4**	**2**	**1**

Many possible answers **Row 2 correct** **B1**

Rest correct **B1**

4. **Either**

4	2	3	**1**
		2	4
		4	2
2	4	1	**3**

Or

4	2	3	1
		2	**4**
		4	**2**
2	4	1	3

B2

5. In the top row there are 9 ways of allocating a symbol to the left cell, then 8 for the next, 7 for the next and so on down to 1 for the right cell, giving **M1**

9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 9! ways. **E1**

So there must be 9!× the number of ways of completing the rest of the puzzle.

6.

(i)

Block side length, b	**Sudoku,** $s \times s$	M
1	1×1	-
2	4×4	12
3	9×9	**77**
4	16×16	**252**
5	**25×25**	**621**

25×25 **B1**

77, 252 and 621 **B1**

(ii) $M = b^4 - 4$

b^4 **B1**

- 4 **B1**

7.		
(i)	There are neither 3s nor 5s among the givens.	M1
	So they are interchangeable and therefore there is no unique solution	E1
(ii)	The missing symbols form a 3×3 embedded Latin square.	M1
	There is not a unique arrangement of the numbers 1, 2 and 3 in this square.	E1
		[18]