

General Certificate of Education
January 2009
Advanced Level Examination



MATHEMATICS
Unit Pure Core 3

MPC3

Monday 19 January 2009 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Question 3 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

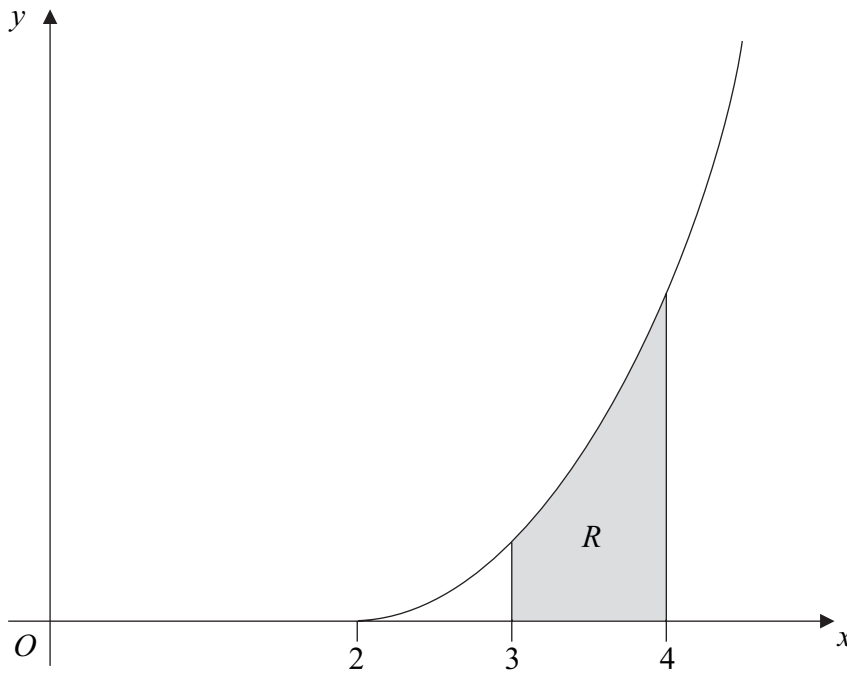
Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to $\int_1^9 \frac{1}{1 + \sqrt{x}} dx$, giving your answer to three significant figures. (4 marks)

- 2 The diagram shows the curve with equation $y = \sqrt{(x - 2)^5}$ for $x \geq 2$.



The shaded region R is bounded by the curve $y = \sqrt{(x - 2)^5}$, the x -axis and the lines $x = 3$ and $x = 4$.

Find the exact value of the volume of the solid formed when the region R is rotated through 360° about the x -axis. (4 marks)

3 [Figure 1, printed on the insert, is provided for use in this question.]

The curve with equation $y = x^3 + 5x - 4$ intersects the x -axis at the point A , where $x = \alpha$.

(a) Show that α lies between 0.5 and 1. (2 marks)

(b) Show that the equation $x^3 + 5x - 4 = 0$ can be rearranged into the form

$$x = \frac{1}{5}(4 - x^3) \quad (1 \text{ mark})$$

(c) Use the iteration $x_{n+1} = \frac{1}{5}(4 - x_n^3)$ with $x_1 = 0.5$ to find x_3 , giving your answer to three decimal places. (2 marks)

(d) The sketch on **Figure 1** shows parts of the graphs of $y = \frac{1}{5}(4 - x^3)$ and $y = x$, and the position of x_1 .

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x -axis. (2 marks)

4 (a) Solve the equation $\sec x = \frac{3}{2}$, giving all values of x to the nearest degree in the interval $0^\circ < x < 360^\circ$. (2 marks)

(b) By using a suitable trigonometrical identity, solve the equation

$$2 \tan^2 x = 10 - 5 \sec x$$

giving all values of x to the nearest degree in the interval $0^\circ < x < 360^\circ$. (6 marks)

Turn over for the next question

Turn over ►

5 The functions f and g are defined with their respective domains by

$$f(x) = 2 - x^4 \quad \text{for all real values of } x$$

$$g(x) = \frac{1}{x-4} \quad \text{for real values of } x, \ x \neq 4$$

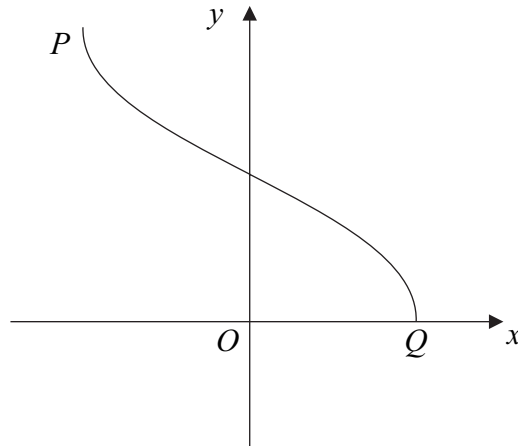
- (a) State the range of f . *(2 marks)*
- (b) Explain why the function f does not have an inverse. *(1 mark)*
- (c) (i) Write down an expression for $fg(x)$. *(1 mark)*
- (ii) Solve the equation $fg(x) = -14$. *(3 marks)*

6 A curve has equation $y = e^{2x}(x^2 - 4x - 2)$.

- (a) Find the value of the x -coordinate of each of the stationary points of the curve. *(6 marks)*
- (b) (i) Find $\frac{d^2y}{dx^2}$. *(2 marks)*
- (ii) Determine the nature of each of the stationary points of the curve. *(2 marks)*

- 7 (a) Given that $3e^x = 4$, find the exact value of x . *(2 marks)*
- (b) (i) By substituting $y = e^x$, show that the equation $3e^x + 20e^{-x} = 19$ can be written as $3y^2 - 19y + 20 = 0$. *(1 mark)*
- (ii) Hence solve the equation $3e^x + 20e^{-x} = 19$, giving your answers as exact values. *(3 marks)*

8 The sketch shows the graph of $y = \cos^{-1} x$.



- (a) Write down the coordinates of P and Q , the end points of the graph. (2 marks)
- (b) Describe a sequence of two geometrical transformations that maps the graph of $y = \cos^{-1} x$ onto the graph of $y = 2 \cos^{-1}(x - 1)$. (4 marks)
- (c) Sketch the graph of $y = 2 \cos^{-1}(x - 1)$. (2 marks)
- (d) (i) Write the equation $y = 2 \cos^{-1}(x - 1)$ in the form $x = f(y)$. (2 marks)
- (ii) Hence find the value of $\frac{dx}{dy}$ when $y = 2$. (3 marks)

- 9 (a) Given that $y = \frac{4x}{4x - 3}$, use the quotient rule to show that $\frac{dy}{dx} = \frac{k}{(4x - 3)^2}$, where k is an integer. (2 marks)
- (b) (i) Given that $y = x \ln(4x - 3)$, find $\frac{dy}{dx}$. (3 marks)
- (ii) Find an equation of the tangent to the curve $y = x \ln(4x - 3)$ at the point where $x = 1$. (3 marks)
- (c) (i) Use the substitution $u = 4x - 3$ to find $\int \frac{4x}{4x - 3} dx$, giving your answer in terms of x . (4 marks)
- (ii) By using integration by parts, or otherwise, find $\int \ln(4x - 3) dx$. (4 marks)

END OF QUESTIONS

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