4767

Mark Scheme

C	Question	Answer		Guidance
1	(i)	2.5 1.5 1.5 0.5 0 0 20 40 60 80 100 Thickness	G1 G2,1,0	G1 For axes suitably labelled with some indication of linear scale provided.Allow x & y Allow axes reversed.G2 for points plotted correctly. G1 if 4 points plotted correctly. G0 if two or more incorrectly plotted/omitted points.Special Case SC1 for points visibly correct on axes where no indication of scale has been provided.
			[3]	
1	(ii)	Thickness is the independent variable since the values of 'Thickness' are not subject to random variation, but are determined by the manufacturer.	E1	Allow explanations referring to thickness being "controlled" by the manufacturer. Allow equivalent interpretations.
1	(iii)	$\overline{4}$ (0) $\overline{1}$ 1549	[1] B1	For t and t word SOL
	()	$l = 00, \ n = 1.548$		(e.g. can be implied by $b = 0.0206$)
		$b = \frac{S_{th}}{S_{tt}} = \frac{546.8 - (300 \times 7.74 / 5)}{22000 - 300^2 / 5} = \frac{82.4}{4000} =$	M1*	For attempt at calculating gradient (b) for h on t .
		0.0206	A1	For 0.0206 cao
		OR $b = \frac{546.8 / 5 - (60 \times 1.548)}{22000 / 5 - 60^2} = \frac{16.48}{800} = 0.0206$ hence least squares regression line is: $h - \bar{h} = b(t - \bar{t})$		

C	Question		Answer		Guidance	
			$\Rightarrow h - 1.548 = 0.0206 (t - 60)$	M1	For equation of line, using their $b, b > 0$, and	
				dep*	passing through their (\bar{t}, \bar{h})	
			$\Rightarrow h = 0.0206 t + 0.312$	A1	Final equation must have h as the subject.	
					Allow $h = 0.021 t + 0.31$,	
					Allow $h = 0.021 t + 0.288$	
					unless $x & y$ defined appropriately	
				[5]		
1	(iv)	(A)	$(0.0206 \times 70) + 0.312 = 1.754$	B1	Allow 1.75	
					FT their equation provided $b > 0$	
			Likely to be reliable as interpolation	E1		
				[2]		
1	(iv)	(B)	$(0.0206 \times 120) + 0.312 = 2.784$	B1	Allow 2.78	
					FT their equation provided $b > 0$	
			Could be unreliable as extrapolation	E1	Condone "reliable as 120 is not too far away from	
					the data used to produce the equation"	
				[2]		
1	(v)		Thickness = $40 \implies$ predicted max height			
			$= (0.0206 \times 40) + 0.312 = 1.136$	M1	For prediction. FT their equation provided $b > 0$	
			Residual = 1.09 - 1.136	M1	For difference between 1.09 and prediction.	
			= -0.046	A1	Allow –0.05	
				[3]		
1	(vi)		Regression line gives a prediction of	B1*	B1 for obtaining a prediction from regression	
			$(0.0206 \times 200) + 0.312 = 4.432$		equation or from graph	
			This is well above the observed value.	El	El for noting the large difference between	
				dep*	F1 for witchle intermetation man line th	
			It could be that the relationship breaks down for	EI	E1 for suitable interpretation regarding the	
			arger unekness, of that the relationship is not inlear	[3]	relationship between maximum nergitt and unckness	
		1		[ວ]		

Question		ion	Answer		Guidance	
2	(i)	(A)	$P(X = 0) = e^{-2.1} 2.1^0$			
			P(X = 0) = -0!	M1	For calculation	
			= 0.1225	A1	CAO Allow 0.122	
			Or from tables $P(X = 0) = 0.1225$			
				[2]		
2	(i)	(B)	$P(X \ge 2) = 1 - P(X \le 1) = 1 - 0.3796$	M1	M1 for use of correct structure. i.e. M0 for use of $1 - P(X \le 2)$ or $1 - 0.6796$	
					Using $\lambda = 2.0$ leading to $1 - 0.4060$ gets M1	
			= 0.6204	A1	CAO Allow 0.6203, 0.620	
				[2]		
2	(i)	(<i>C</i>)	New $\lambda = 5 \times 2.1 = 10.5$	B1	For mean (SOI)	
			P(Between 5 and 10 in 5 mins)			
			= 0.5207 - 0.0211	M1	For $P(X \le 10) - P(X \le 4)$ used.	e.g. 1 – 0.9379
						BOM1A0
			= 0.4996	A1	CAO Allow 0.500, 0.50, Condone 0.5 www.	Dominio
				[3]		
2	(ii)		Mean number in 60 minutes = $60 \times 2.1 = 126$			
			Using Normal approx. to the Poisson,	B1	For Normal approx.	
			$X \sim N(126, 126)$	B1	For correct parameters (SOI)	
			$P(X \ge 130) = P\left(Z \ge \frac{129.5 - 126}{\sqrt{126}}\right)$	B1	For correct continuity correction	
			$= P(Z > 0.3118) = 1 - \Phi(0.3118)$	M1	For correct probability structure	
			= 1 - 0.6224			
			= 0.3776	A1	CAO, (Do not FT wrong or omitted CC).	
					Allow 0.378www & 0.3775	
				[5]		

Question		Answer	Marks	Guidance	
2 (iii)	(Because if butterflies are blown in pairs,) the events will no longer be occurring singly.	E1		Accept 'not independent'
			[1]		
2 (iv)	P(3 or fewer) = P(3 or fewer individuals and no pairs) +			
		P(0 or 1 individual and 1 pair)	M1	For P(0 pairs) (= 0.8187)	
		$= (0.9068 \times 0.8187) + (0.4932 \times (0.9825 - 0.8187))$	M1	For P(1 pair) (= 0.1638 or 0.1637)	First two M1s can be awarded for 0.9825
		$= (0.9068 \times 0.8187) + (0.4932 \times 0.1638)$	M2	For structure M2 for correct 6 combinations identified and their probabilities added, M1 for 5 combinations identified and their probabilities added.	
		= 0.7424 + 0.0808			
		= 0.8232	A1	CAO Allow awrt 0.823	
		Or			
		using <i>D</i> for the number of pairs and <i>S</i> for the number of singles $P(D = 0) \times P(S = 0) = e^{-0.2} \times e^{-1.7} = 0.1495$ $P(D = 0) \times P(S = 1) = e^{-0.2} \times 1.7e^{-1.7} = 0.2542$ $P(D = 0) \times P(S = 2) = e^{-0.2} \times 1.7^{2}e^{-1.7} \div 2 = 0.2161$ $P(D = 0) \times P(S = 3) = e^{-0.2} \times 1.7^{3}e^{-1.7} \div 3! = 0.1224$ $P(D = 1) \times P(S = 0) = 0.2e^{-0.2} \times e^{-1.7} = 0.0299$ $P(D = 1) \times P(S = 1) = 0.2e^{-0.2} \times 1.7e^{-1.7} = 0.0508$			
		Or			
		$P(D = 0) \times P(S = 3) + P(D = 1) \times P(S = 1)$ $P(D = 0) \times P(S = 2) + P(D = 1) \times P(S = 0)$ $P(D = 0) \times P(S = 1)$ $P(D = 0) \times P(S = 0)$		P(3 butterflies) (= 0.1733) P(2 butterflies) (= 0.2460) P(1 butterfly) (= 0.2542) P(0 butterflies) (= 0.1495)	

C	luest	ion	Answer		Guidance
			Or $P(D \le 1) \times P(S \le 1) = 0.98247 \times 0.49324$ $D(D = 0) \times P(S = 2) = 0.21613$ $D(D = 0) \times P(S = 3) = 0.12247$	[5]	
3	(i)	(A)	$P(X < 450) = P\left(Z < \frac{450 - 435}{30}\right)$	M1	For standardising. M0 if 'continuity correction' applied
			$= P(Z < 0.5) = \Phi(0.5)$ = 0.6915	M1 A1 [3]	For correct structure CAO Allow 0.692
3	(i)	(B)	P(400 < X < 450) = $P\left(\frac{400 - 435}{30} < Z < \frac{450 - 435}{30}\right)$ = $P(-1.1667 < X < 0.5)$ = $\Phi(0.5) - \Phi(-1.1667)$ = $0.6915 - 0.1216$	M1 B1	For correct structure For use of difference column to obtain 0.8784,
			= 0.5699	A1 [3]	0.8783, 0.1216 or 0.1217. Condone 0.8782 or 0.1218 FT "their 0.6915" – 0.1216 (or 0.1217)
3	(ii)		P(all 5 between 400 and 450) = 0.5699 ⁵ = 0.0601	M1 A1 [2]	FT Allow 0.060

Question		ion	Answer		Guidance		
3	(iii)		$P(Y < 350) = 0.2, P(Y > 390) = 0.1$ $P(Z < \frac{350 - \mu}{\sigma}) = 0.2$ $\Phi^{-1}(0.2) = -0.8416$ $\frac{350 - \mu}{\sigma} = -0.8416$ $P(Z > \frac{390 - \mu}{\sigma}) = 0.1$	M1	For equation as seen or equivalent with their –ive z value	If 'continuity corrections' applied allow M marks but do not award final A marks	
			$\Phi^{-1}(0.9) = 1.282$	B1	For 1.282 or -0.8416		
			$\frac{390 - \mu}{\sigma} = 1.282$ $350 = \mu = 0.8416 \sigma$	M1	For equation as seen or equivalent with their +ive z value		
			$390 = \mu + 1.282 \sigma$				
			$2.1236 \sigma = 40$				
			$\sigma = 18.84$	A1	Allow 18.8	Answers to max 2 d.p.	
			$\mu = 350 + (0.8416 \times 18.84) = 365.85$	A1	Allow 365.86, 366, 365.9		
				[5]			
3	(iv)		$\Phi^{-1}(0.975) = 1.96$	B1	For using a suitable pair of z values e.g. ± 1.96		
			$a = 365.85 - (1.96 \times 18.84)$	M1	For either equation provided that a suitable pair of <i>z</i> -values is used. e.g. $+2.326$ and -1.751		
			= 328.9		FT their μ and σ to 2 d.p. (A0 if 'continuity	Accept any	
				A1	correction' used)	correct values of <i>a</i> and <i>b</i> .	
			$b = 365.85 + (1.96 \times 18.84)$				
			= 402.8	A1	FT their μ and σ to 2 d.p. (A0 if 'continuity correction' used)		
				[4]			

Question		on	Answer			Guidance		
4 (a)		H ₀ : no associ H ₁ : some asso	H ₀ : no association between grade and hours worked H ₁ : some association between grade and hours worked;			Hypotheses in context		
			Less than 5hrs	At least 5hrs				
		A or B	17.05	13.95	M1	Any row/column correct		
		C or lower	15.95	13.05	AI	For expected values (to 2 dp)		
			·				NB These	
			Less than 5hrs	At least 5hrs	M1	For valid attempt at (O-E) ² /E. Any row column	M1A1 marks	
		A or B	0.5104	0.6238	A1	correct. ca For all correct im contract contract	cannot be	
		C or lower	0.5456	0.6669	1		correct final	
			i				value of X^2	
		$X^2 = 2.347$			B1			
		Refer to χ_1^2			M1	M1 For 1 deg of freedom. No FT from here if wrong. A1 CAO for cv or p -value = 0.1255. SC1 for cv or p -value if 1 dof not seen.		
		Critical value	e at 5% level = 3.84	1	A1			
		Result is not	Result is not significant.					
		There is insut association b	fficient evidence to etween hours worke	suggest that there and grade.	is any E1	For conclusion in context. NB if $H_0 H_1$ reversed, or 'correlation' mentioned, do not award first B1 or final E1		
					[9]			

C	Question		Answer		Guidance
4	(b)		$\overline{x} = 417.79$	B1	For \overline{x}
			$H_0: \mu = 420;$	B1	For use of 420 in hypotheses. Hypotheses in words must refer to population. Do not allow alternative symbols unless clearly defined as the population mean.
			$H_1: \mu \neq 420$	B1	For both correct
			Where μ denotes the mean volume of the cans of tomato purée (in the population)	B1	For definition of μ . Condone omission of "population" if correct notation μ is used, but if μ is defined as the sample mean then award B0 .
			Test statistic $=$ $\frac{417.79 - 420}{3.5 / \sqrt{10}} = \frac{-2.21}{1.107} = -1.997$	M1* A1	must include $\sqrt{10}$ FT their $\bar{\mathbf{x}}$
			Lower 1% level 2 tailed critical value of $z = -2.576$	B1*	For -2.576 Must be -2.576 unless it is clear that absolute values are being used.
			-1.997 > -2.576	M1 dep*	For sensible comparison leading to a conclusion.
			So not significant. There is insufficient evidence to reject H0	Ĩ	
			There is insufficient evidence to suggest that the average volumes of the cans of tomato purée is not 420ml	A1	For conclusion in words in context provided that correct cv used. FT only candidate's test statistic.
				[9]	

ADDITIONAL NOTES REGARDING QUESTION 4 (b)

<u>Critical Value Method</u> $420 - 2.576 \times 3.5 \div \sqrt{10}$ gets M1*B1* = 417.148... gets A1 417.79 > 417.148.. gets M1dep* for sensible comparison A1 still available for correct conclusion in words & context

Confidence Interval Method CI centred on 417.79 + or $-2.5756 \times 3.5 \div \sqrt{10}$ gets M1* B1* = (414.93..., 420.64..) gets A1 NOTE that the final M1dep* A1 available only if 2.576 used. "Contains 420" gets M1dep* A1 still available for correct conclusion in words & context

Probability Method

Finding P(sample mean < 417.79) = 0.0229 gets M1* A1 B1*

 $0.0229 > 0.005^*$ gets M1dep* for a sensible comparison if a conclusion is made.

A1 available for a correct conclusion in words & context.

Condone P(sample mean > 417.79) = 0.9771 for M1* but only allow A1 B1* if sensible comparison made, at which point the final M1dep* and A1 are still available

ADDITIONAL NOTE REGARDING OVER-SPECIFICATION OF ANSWERS

Over-specification by providing final answers correct to 5 or more significant figures will be penalised. When this applies, candidates may lose no more than 2 marks per question and no more than 4 marks in total. The only exception to this rule is in Question 3 parts (iii) & (iv) – see guidance notes.