Mark Scheme 4754
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Section A

1 $\frac{2x}{x-2}-\frac{4x}{x+1}=3$ $\Rightarrow$ $2x(x+1)-4x(x-2)=3(x-2)(x+1)$ $\Rightarrow$ $2x^2+2x-4x^2+8x=3x^2-3x-6$ $\Rightarrow$ $0=5x^2-13x-6$ $=(5x+2)(x-3)$ $\Rightarrow$ $x=-2/5$ or 3.	M1 M1 A1 M1 A1 cao [5]	Clearing fractions expanding brackets oe factorising or formula
2 $dx/dt = 1 - 1/t$ $dy/dt = 1 + 1/t$ $\Rightarrow$ $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$ $=\frac{1+\frac{1}{t}}{1-\frac{1}{t}}$ When $t = 2$, $dy/dx = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = 3$	B1 M1 A1 M1 A1 [5]	Either dx/dt or dy/dt soi www
3 $\overrightarrow{BA}=\begin{pmatrix}-4\\1\\-3\end{pmatrix}$, $\overrightarrow{BC}=\begin{pmatrix}2\\5\\-1\end{pmatrix}$ $\overrightarrow{BA}.\overrightarrow{BC}=\begin{pmatrix}-4\\1\\-3\end{pmatrix}.\begin{pmatrix}2\\5\\-1\end{pmatrix}=(-4)\times 2+1\times 5+(-3)\times(-1)$ $= -8 + 5 + 3 = 0$ $\Rightarrow$ angle ABC = 90° Area of triangle = ½ × BA × BC $=\frac{1}{2}\times\sqrt{(-4)^2+1^2+3^2}\times\sqrt{2^2+5^2+(-1)^2}$ $= \frac{1}{2} \times \sqrt{26} \times \sqrt{30}$ = 13.96 sq units	B1 M1 A1 M1 M1 A1 [6]	soi , condone wrong sense scalar product = 0 area of triangle formula oe length formula accept 14.0 and √195

4(i) $2\sin 2\theta + \cos 2\theta = 1$ $\Rightarrow$ $4\sin\theta\cos\theta + 1 - 2\sin^2\theta = 1$ $\Rightarrow 2\sin\theta(2\cos\theta - \sin\theta) = 0$ or $4\tan\theta - 2\tan^2\theta = 0$ $\Rightarrow$ $\sin\theta = 0$ or $\tan\theta = 0$, $\theta = 0°, 180°$ or $2\cos\theta - \sin\theta = 0$ $\Rightarrow$ $\tan\theta = 2$ $\Rightarrow$ $\theta = 63.43°, 243.43°$ OR Using $R\sin(2\theta+\alpha)$ $R=\sqrt{5}$ and $\alpha=26.57°$ $2\theta + 26.57 = \arcsin 1/R$ $\theta=0°, 180°$ $\theta=63.43°, 243.43°$	M1 A1 A1 M1 A1, A1 [6] M1 A1 M1 A1 A1,A1 [6]	Using double angle formulae Correct simplification to factorisable or other form that leads to solutions 0° and 180° $\tan\theta = 2$ (-1 for extra solutions in range) (-1 for extra solutions in range)
5 (i) Plane has equation $x - y + 2z = c$ At $(2, -1, 4)$, $2 + 1 + 8 = c$ $\Rightarrow c = 11$.	B1 M1 A1	$x - y + 2z = c$ finding c
(ii) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7+\lambda \\ 12+3\lambda \\ 9+2\lambda \end{pmatrix}$ $\Rightarrow$ $7 + \lambda - (12 + 3\lambda) + 2(9 + 2\lambda) = 11$ $\Rightarrow$ $2\lambda = -2$ $\Rightarrow$ $\lambda = -1$ Coordinates are $(6, 9, 7)$	M1 M1 A1 A1 [7]	 ft their equation from (i) ft their x-y+2z=c cao
6 (i) $\dfrac{1}{\sqrt{4-x^2}} = 4^{-\frac{1}{2}}(1-\frac{1}{4}x^2)^{-\frac{1}{2}}$ $= \frac{1}{2}[1 + (-\frac{1}{2})(-\frac{1}{4}x^2) + \dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\frac{1}{4}x^2)^2 + ...]$ $= \frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4 + ...$	M1 M1 A1 A1	 Binomial coeffs correct Complete correct expression inside bracket cao
(ii) $\int_0^1 \dfrac{1}{\sqrt{4-x^2}}dx \approx \int_0^1 (\frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4)dx$ $= \left[\frac{1}{2}x + \frac{1}{48}x^3 + \frac{3}{1280}x^5\right]_0^1$ $= \frac{1}{2} + \frac{1}{48} + \frac{3}{1280}$ $= 0.5232$ (to 4 s.f.)	M1ft A1	
(iii) $\int_0^1 \dfrac{1}{\sqrt{4-x^2}}dx = \left[\arcsin\frac{x}{2}\right]_0^1$ $= \pi/6 = 0.5236$	B1 [7]	

Section B

7(i) $\mathrm{A}\hat{\mathrm{O}}\mathrm{P} = 180 - \beta = 180 - \alpha - \theta$ $\Rightarrow \quad \beta = \alpha + \theta$ $\Rightarrow \quad \theta = \beta - \alpha$ $\tan\theta = \tan(\beta - \alpha)$ $= \dfrac{\tan\beta - \tan\alpha}{1 + \tan\beta\tan\alpha}$ $= \dfrac{\frac{y}{10} - \frac{y}{16}}{1 + \frac{y}{10}.\frac{y}{16}}$ $= \dfrac{16y - 10y}{160 + y^2}$ $= \dfrac{6y}{160 + y^2}$ * When $y = 6$, $\tan\theta = 36/196$ $\Rightarrow \quad \theta = 10.4°$	M1 M1 E1 M1 A1 E1 M1 A1 cao [8]	Use of sum of angles in triangle OPT and AOP oe SC B1 for $\beta=\alpha+\theta$, $\theta=\beta-\alpha$ no justification Use of Compound angle formula Substituting values for tan α and tan β www accept radians
(ii) $\sec^2\theta\dfrac{d\theta}{dy} = \dfrac{(160 + y^2)6 - 6y.2y}{(160 + y^2)^2}$ $= \dfrac{6(160 + y^2 - 2y^2)}{(160 + y^2)^2}$ $\Rightarrow \quad \dfrac{d\theta}{dy} = \dfrac{6(160 - y^2)}{(160 + y^2)^2}\cos^2\theta$ *	M1 M1 A1 A1 E1 [5]	$\sec^2\theta\dfrac{d\theta}{dy} = \ldots$ quotient rule correct expression simplifying numerator www
(iii) $d\theta/dy = 0$ when $160 - y^2 = 0$ $\Rightarrow \quad y^2 = 160$ $\Rightarrow \quad y = 12.65$ When $y = 12.65$, $\tan\theta = 0.237\ldots$ $\Rightarrow \quad \theta = 13.3°$	M1 A1 M1 A1cao [4]	oe accept radians

8 (i) $x = a(1 + kt)^{-1}$ $\Rightarrow$ $dx/dt = -ka(1 + kt)^{-2}$ $= -ka(x/a)^2$ $= -kx^2/a$ * OR $kt = a/x - 1$, $t = a/kx - 1/k$ $dt/dx = -a/kx^2$ $\Rightarrow dx/dt = -kx^2/a$	M1 A1 E1 [3] M1 A1 E1 [3]	Chain rule (or quotient rule) Substitution for x
(ii) When $t = 0$, $x = a \Rightarrow a = 2.5$ When $t = 1$, $x = 1.6 \Rightarrow 1.6 = 2.5/(1 + k)$ $\Rightarrow$ $1 + k = 1.5625$ $\Rightarrow$ $k = 0.5625$	B1 M1 A1 [3]	$a = 2.5$
(iii) In the long term, $x \to 0$	B1 [1]	or, for example, they die out.
(iv) $\frac{1}{2y-y^2} = \frac{1}{y(1-y)} = \frac{A}{y} + \frac{B}{2-y}$ $\Rightarrow$ $1 = A(2 - y) + By$ $y = 0 \Rightarrow 2A = 1 \Rightarrow A = ½$ $y = 2 \Rightarrow 1 = 2B \Rightarrow B = ½$ $\Rightarrow \frac{1}{2y-y^2} = \frac{1}{2y} + \frac{1}{2(2-y)}$	M1 M1 A1 A1 [4]	partial fractions evaluating constants by substituting values, equating coefficients or cover-up
(v) $\int \frac{1}{2y-y^2}dy = \int dt$ $\Rightarrow$ $\int [\frac{1}{2y} + \frac{1}{2(2-y)}]dy = \int dt$ $\Rightarrow$ $½ \ln y - ½ \ln(2 - y) = t + c$ When $t = 0$, $y = 1 \Rightarrow 0 - 0 = 0 + c \Rightarrow c = 0$ $\Rightarrow$ $\ln y - \ln(2 - y) = 2t$ $\Rightarrow$ $\ln \frac{y}{2-y} = 2t$ * $\frac{y}{2-y} = e^{2t}$ $\Rightarrow$ $y = 2e^{2t} - ye^{2t}$ $\Rightarrow$ $y + ye^{2t} = 2e^{2t}$ $\Rightarrow$ $y(1 + e^{2t}) = 2e^{2t}$ $\Rightarrow$ $y = \frac{2e^{2t}}{1+e^{2t}} = \frac{2}{1+e^{-2t}}$ *	M1 B1 ft A1 E1 M1 DM1 E1 [7]	Separating variables $½ \ln y - ½ \ln(2 - y)$ ft their A,B evaluating the constant Anti-logging Isolating y
(vi) As $t \to \infty$ $e^{-2t} \to 0 \Rightarrow y \to 2$ So long term population is 2000	B1 [1]	or $y = 2$

Comprehension

1. It is the largest number in the Residual column in Table 5. **B1**

2. (i)

Acceptance percentage, *a*%		10%	14%	**12%**	**11%**	**10.5%**
Party	**Votes (%)**	**Seats**	**Seats**	**Seats**	**Seats**	**Seats**
P	30.2	**3**	**2**	**2**	**2**	**2**
Q	11.4	**1**	**0**	**0**	**1**	**1**
R	22.4	**2**	**1**	**1**	**2**	**2**
S	14.8	**1**	**1**	**1**	**1**	**1**
T	10.9	**1**	**0**	**0**	**0**	**1**
U	10.3	**1**	**0**	**0**	**0**	**0**
Total seats		**9**	**4**	**4**	**6**	**7**

Seat Allocation P 2 Q 1 R 2 S 1 T 1 U 0

10% & 14% B1
Trial M1
10.5% (10.3<x≤10.9) A1
Allocation A1

(ii)

	Round							
Party	1	2	3	4	5	6	7	Residual
P	30.2	**15.1**	**15.1**	**10.07**	**10.07**	**10.07**	**10.07**	**10.07**
Q	11.4	**11.4**	**11.4**	**11.4**	**11.4**	**5.7**	**5.7**	**5.7**
R	22.4	**22.4**	**11.2**	**11.2**	**11.2**	**11.2**	**7.47**	**7.47**
S	14.8	**14.8**	**14.8**	**14.8**	**7.4**	**7.4**	**7.4**	**7.4**
T	10.9	**10.9**	**10.9**	**10.9**	**10.9**	**10.9**	**10.9**	**5.45**
U	10.3	**10.3**	**10.3**	**10.3**	**10.3**	**10.3**	**10.3**	**10.3**
Seat allocated to	**P**	**R**	**P**	**S**	**Q**	**R**	**T**	

Seat Allocation P 2 Q 1 R 2 S 1 T 1 U 0

General method **M1** Round 2 correct **A1** Round 5 correct **A1**(condone minor arithmetic error) Residuals **A1** www Allocation **A1**cso

3. $\frac{11.2}{1+1} < 11 \leq \frac{11.2}{1} \Rightarrow 5.6 < 11 \leq 11.2$

M1, A1

for either or both
M1 only for 5.6<a≤11.2

4. (i) The end-points of the intervals are the largest values in successive columns of Table 5.(or two largest within a column)

B1

So in

2	$16.6 < a \leq 22.2$

22.2 is the largest number in Round 2. 16.6 is the largest number in Round 3.

B1

(ii)

Seats	a		**Seats**	a
1	$22.2 < a \leq 27.0$		5	$11.1 < a \leq 11.2$
2	$16.6 < a \leq 22.2$		6	$10.6 < a \leq 11.1$
3	$13.5 < a \leq 16.6$		7	$9.0 < a \leq 10.6$
4	$11.2 < a \leq 13.5$			

5. (i) ● means $\leq$, ○ means $<$ (greater or less than)

B1

(ii) $\frac{V_k}{N_k+1} < a$ $\quad a \leq \frac{V_k}{N_k}$

$V_k < aN_k + a$ $\quad aN_k \leq V_k$

$V_k - aN_k < a$ $\quad 0 \leq V_k - aN_k$

$0 \leq V_k - aN_k < a$

B1

(iii) The unused votes may be zero but must be less than a. **B1**