OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **4724**

Core Mathematics 4

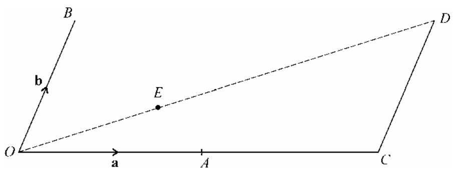
MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

This mark scheme consists of 4 printed pages.

 [Turn over

1	$\dfrac{x^4+1}{x^2+1}=x^2-1+\dfrac{2}{x^2+1}$	B1		For correct leading term x^2 in quotient
		M1		For evidence of correct division process
		A1		For correct quotient x^2-1
		A1	4	For correct remainder 2
			[4]	
2	**(i)** $(1-2x)^{-\frac{1}{2}}=1+(-\tfrac{1}{2})(-2x)+\dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-2x)^2+$			
	$+\dfrac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}(-2x)^3+\dots$	M1		For 2nd, 3rd or 4th term OK (unsimplified)
	$=1+x+\tfrac{3}{2}x^2+\tfrac{5}{2}x^3$	A1		For $1+x$ correct
		A1		For $+\tfrac{3}{2}x^2$ correct
		A1	4	For $+\tfrac{5}{2}x^3$ correct
	(ii) Valid for $\lvert x\rvert<\tfrac{1}{2}$	B1	1	For any correct expression(s)
			[5]	
3	$\int_0^1 x\,\mathrm{e}^{-2x}\,\mathrm{d}x=\left[-\tfrac{1}{2}x\,\mathrm{e}^{-2x}\right]_0^1-\int_0^1-\tfrac{1}{2}\mathrm{e}^{-2x}\,\mathrm{d}x$	M1		For attempt at 'parts' going the correct way
		A1		For correct terms $-\tfrac{1}{2}x\,\mathrm{e}^{-2x}-\int-\tfrac{1}{2}\mathrm{e}^{-2x}\,\mathrm{d}x$
	$=\left[-\tfrac{1}{2}x\,\mathrm{e}^{-2x}-\tfrac{1}{4}\mathrm{e}^{-2x}\right]_0^1$	M1		For consistent attempt at second integration
		M1		For correct use of limits throughout
	$=\tfrac{1}{4}-\tfrac{3}{4}\mathrm{e}^{-2}$	A1	5	For correct (exact) answer in any form
			[5]	
4	**(i)** 	B1		For C correctly located on sketch
		B1		For D correctly located on sketch
		B1√	3	For E correctly located wrt O and D
	(ii) $\overrightarrow{AE}=\tfrac{1}{3}(2\mathbf{a}+\mathbf{b})-\mathbf{a}=\tfrac{1}{3}(\mathbf{b}-\mathbf{a})$	M1		For relevant subtraction involving $\overrightarrow{OE}$
		A1		For correct expression for $(\pm)\overrightarrow{AE}$ or $\overrightarrow{EB}$
	Hence *AE* is parallel to *AB*	A1		For correct recognition of parallel property
	i.e. *E* lies on the line joining *A* to *B*	A1	4	For complete proof of required result
			[7]	
5	**(i)** $4x+x\dfrac{\mathrm{d}y}{\mathrm{d}x}+y+2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$	B1		For correct terms $x\dfrac{\mathrm{d}y}{\mathrm{d}x}+y$
		B1		For correct term $2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$
	Hence $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{4x+y}{x+2y}$	M1		For solving for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$
		A1	4	For any correct form of expression
	(ii) $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow y=-4x$	M1		For stating or using their $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$
	Hence $2x^2+(-4x^2)+(-4x)^2=14$	M1		For solving simultaneously with curve equn
	i.e. $x^2=1$	A1		For correct value of x^2 (or y^2)
	So the two points are $(1,-4)$ and $(-1,4)$	A1	4	For both correct points identified
			[8]	

6 **(i)**	$\theta = 0$ at the origin A is $(0, a\pi)$ B is $(a, 0)$	B1 B1 B1 **3**	For the correct value For the correct y-coordinate at A For the correct x-coordinate at B
(ii)	$\frac{dx}{d\theta} = a\cos\theta$ $\frac{dy}{d\theta} = a(\cos\theta - \theta\sin\theta)$ Hence $\frac{dy}{dx} = \frac{\cos\theta - \theta\sin\theta}{\cos\theta} = 1 - \theta\tan\theta$ Gradient of tangent at the origin is 1 Hence equation is $y = x$	B1 M1 M1 A1 M1 A1 **6** **9**	For correct differentiation of x For differentiating y using product rule For use of $\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta}$ For given result correctly obtained For using $\theta = 0$ For correct equation
6 **(i)**	$L_1 : \mathbf{r} = 3\mathbf{i} + 6\mathbf{j} + \mathbf{k} + s(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$ $L_2 : \mathbf{r} = 3\mathbf{i} - \mathbf{j} + 4\mathbf{k} + t(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	M1 A1 **2**	For correct RHS structure for either line For both lines correct
(ii)	$3 + 2s = 3 + t,\ 6 + 3s = -1 - 2t,\ 1 - s = 4 + t$ First pair of equations give $s = -1, t = -2$ Third equation checks: $1 + 1 = 4 - 2$ Point of intersection is $(1, 3, 2)$	M1 M1 A1 A1 A1 **5**	For at least 2 equations with two parameters For solving any relevant pair of equations For both parameters correct For explicit check in unused equation For correct coordinates
(iii)	$2\times1 + 3\times(-2) + (-1)\times1 = (\sqrt{14})(\sqrt{6})\cos\theta$ Hence acute angle is $56.9°$	B1 B1 M1 A1 **4** **11**	For scalar product of correct direction vectors For correct magnitudes $\sqrt{14}$ and $\sqrt{6}$ For correct process for $\cos\theta$ with *any* pair of vectors relevant to these lines For correct acute angle
8 **(i)**	$I = \int \frac{1}{u^2(1+u)^2} \times 2u\ du = \int \frac{2}{u(1+u)^2}\ du$	M1 A1 A1 **3**	For any attempt to find $\frac{dx}{du}$ or $\frac{du}{dx}$ For ‘ $dx = 2u\ du$ ’ or equivalent correctly used For showing the given result correctly
(ii)	$2 \equiv A(1+u)^2 + Bu(1+u) + Cu$ $A = 2$ $C = -2$ $0 = A + B$ (e.g.) $B = -2$	M1 B1 B1 A1 A1 **5**	For correct identity stated For correct value stated For correct value stated For any correct equation involving B For correct value
(iii)	$2\ln u - 2\ln(1+u) + \frac{2}{1+u}$ Hence $I = \ln x - 2\ln(1+\sqrt{x}) + \frac{2}{1+\sqrt{x}} + c$	B1√ B1√ M1 A1 **4** **12**	For $A\ln u + B\ln(1+u)$ with their values For $-C(1+u)^{-1}$ with their value For substituting back For completely correct answer (excluding c)

[Turn over

9	$\frac{dx}{dt} = -k\sqrt{x}$	M1		For use of derivative for rate of change
		A1		For correct equation (neg sign optional here)
	$x = 100$ and $\frac{dx}{dt} = -1 \Rightarrow k = 0.1$	M1		For use of data and their DE to find k
	Hence equation is $\frac{dx}{dt} = -0.1\sqrt{x}$	A1		For any form of correct DE
	$\int x^{-\frac{1}{2}}\,dx = -0.1\int dt \Rightarrow 2x^{\frac{1}{2}} = -0.1t + c$	M1		For separation and integration of both sides
		A1		For $2x^{\frac{1}{2}}$ correct
		A1√		For $(\pm)kt$ correct (the numerical evaluation of k may be delayed until after the DE is solved)
		B1		For one arbitrary constant included (or equivalent statement of both pairs of limits)
	$x = 200, t = 0 \Rightarrow c = 2\sqrt{200}$	M1		For evaluation of c
	So when $x = 100$, $2\sqrt{100} = -0.1t + 2\sqrt{200}$	M1		For evaluation of t
	i.e. $t = 82.8$	A1	**11**	For correct value 82.8 (minutes)
			11	