Mark Scheme (Result)

November 2021

Pearson Edexcel GCE Further Mathematics
Advanced Level in Further Mathematics
Mechanics 1
Paper 9FM0/3C

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

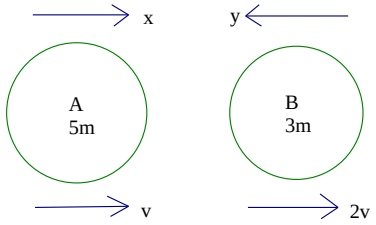
These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

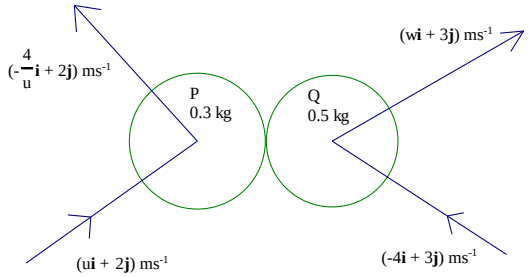
Question	Scheme	Marks	AOs
1(a)	Equation of motion: $F = 500 + 7V$	M1	3.3
	Use of $18000 = F \times V$	M1	3.4
	$\Rightarrow \frac{18000}{V} = 500 + 7V$	A1	1.1b
	$\Rightarrow 7V^2 + 500V - 18000 = 0$	M1	1.1b
	$V = 26 \quad (26.309....)$	A1	1.1b
		(5)	
(b)	Equation of motion:	M1	3.3
	$\frac{18000}{15} - (500 + 7 \times 15) - 900g \times \frac{1}{21} = 900a$	A1 A1	1.1b 1.1b
	$a = 0.194 (0.19) \left(m s^{-2}\right)$	A1	1.1b
		(4)	
			(9 marks)

Notes:	
(a) M1	Dimensionally correct. Condone sign errors. Must be using $a = 0$
M1	Correct use of $P = Fv$
A1	Correct unsimplified equation. Allow with F. Allow with 18K
M1	Form and solve a 3 term quadratic
A1	26 or better (26.309……)
(b)M1	Dimensionally correct. All terms required. Condone sign errors and sin/cos confusion. Omission of g is an accuracy error
A1 **A1**	Unsimplified equation with at most one error Correct unsimplified equation. Allow if $\sin\theta$ not substituted. Allow with 18K
A1	2 sf or 3 sf only not $\frac{7}{36}$

Question	Scheme	Marks	AOs
2(a)			
	Use of CLM	M1	3.1a
	$5mv+6mv\left(=11mv\right)=5mx-3my \qquad \left(11v=5x-3y\right)$	A1	1.1b
	Use of impact law	M1	3.1a
	$v=e\left(x+y\right)$	A1	1.1b
	$\begin{cases}11ev=5ex-3ey\\ 3v=3ex-3ey\end{cases} \Rightarrow x=\frac{v}{8e}\left(11e+3\right)$	M1	3.1a
	$y=\frac{v}{8e}\left(5-11e\right)$	A1	1.1b
	$e>0 \quad \left(\Rightarrow \quad x>0\right) \quad \Rightarrow \quad 5-11e>0$	M1	3.4
	$\Rightarrow 0<e<\frac{5}{11}$	A1	2.2a
		(8)	
(b)	Form equation for KE	M1	2.1
	$\frac{1}{2}\times 5m\times v^2=\frac{16}{100}\times\frac{1}{2}\times 5m\times\frac{v^2}{64e^2}\left(11e+3\right)^2$	A1ft	1.1b
	$\left(4\left(11e+3\right)=\left(\pm\right)80e\right) \qquad e=\frac{1}{3}$	A1	1.1b
	Impulse $=-5m\left(v-x\right)$	M1	3.1a
	$=-5m\left(v-\frac{11v}{8}-\frac{3v}{8e}\right)$ Or: $3m\left(2v+\frac{5v}{8e}-\frac{11v}{8}\right)$	A1ft	1.1b
	Magnitude $=\frac{15}{2}mv$	A1	2.2a
		(6)	
Alt(b)	Form equation for KE	M1	2.1
	$\frac{1}{2}\times 5m\times v^2=\frac{16}{100}\times\frac{1}{2}\times 5m\times x^2$	A1	1.1b
	$\Rightarrow x=\frac{5v}{2},\ y=\frac{v}{2} \quad \Rightarrow e=\frac{1}{3}$	A1	1.1b

	Impulse $= -5m(v - x)$	M1	3.1a
	$= -5m\left(v - \frac{5v}{2}\right)$ Or: $3m\left(2v + \frac{v}{2}\right)$	**A1**	1,16
	Magnitude $= \frac{15}{2}mv$	A1	2.2a
		(6)	
			(14 marks)

Notes:	
(a)M1	All terms required. Dimensionally correct. Condone sign errors
A1	Correct unsimplified equation
M1	Used correctly. Condone sign errors
A1	Correct unsimplified equation
M1	Use their correctly formed equations to solve for v or w or a multiple of v or w
A1	Both velocities correct
M1	Use their velocities (in general form – not by considering one specific value) to form inequality for both moving in the same direction.
A1	Correct only.
(b)M1	Dimensionally correct. Condone 16% on wrong side Allow M or 5m
A1ft	Or equivalent. Correct unsimplified equation. Follow their x Allow M or 5m
A1	Correct answer only Allow M or 5m
M1	Correct use of $I = mv - mu$. Must be subtracting.
A1ft	Accept $\pm$ Follow their x, y, e
A1	Correct only. Must be positive.

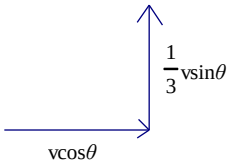
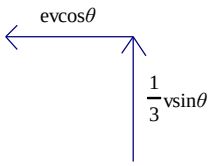
Question	Scheme	Marks	AOs
3(a)			
	For P after: Component in **j** direction $= 2$	B1	3.4
	Deflected through $90°$ so velocity after $= \left(-\frac{4}{u}\mathbf{i} + 2\mathbf{j}\right)\left(\text{m s}^{-1}\right)$	B1	3.4
	CLM parallel to line of centres:	M1	3.1a
	$0.3\left(u + \frac{4}{u}\right) = 0.5(4 + w)$	A1ft	1.1b
	Impact law parallel to line of centres:	M1	3.1a
	$w + \frac{4}{u} = \frac{3}{5}(u + 4)$	A1ft	1.1b
	$\begin{cases} 3u + \frac{12}{u} = 20 + 5w \\ \frac{20}{u} + 5w = 3u + 12 \end{cases}$	M1	1.1b
	$\left(\Rightarrow \frac{32}{u} = 32,\right) \quad u = 1$	A1	2.2a
		(8)	
(b)	For Q after: $w = -1$	B1	1.1b
	$\mathbf{v} = -\mathbf{i} + 3\mathbf{j}$	B1ft	1.1b
	Find relevant angle between directions	M1	3.1a
	$\alpha° = \tan^{-1} 3 - \tan^{-1} \frac{3}{4}$ or $\alpha° = \cos^{-1}\left(\frac{4+9}{5\times\sqrt{10}}\right)$	A1ft	1.1b
	$\alpha = 34.7 \quad (35)$	A1	1.1b
		(5)	
(c)	The line of centres is parallel to the surface the spheres are moving on, so the impulse acts parallel to the surface.	B1	3.5a
		(1)	
			(14 marks)

Notes:	
(a) B1	Correct only Check the diagram
B1	Correct only. Seen or implied.
M1	Correct use of CLM. Need all terms. Condone sign errors
A1ft	Follow their components of velocity of P, with or without a value for the **i** component.
M1	Correct use of the impact law. Condone sign errors
A1ft	Follow their components of velocity of P, with or without a value for the **i** component.
M1	Solve their correctly formed simultaneous equations to obtain value of u.
A1	Correct only
(b) B1	Correct only
B1ft	Follow their w
M1	Correct method to find a relevant angle between the directions
A1ft	Correct unsimplified expression. Follow their **v**
A1	35 or better (34.695…) 0.61 radians
(c)B1	Or equivalent that explains that the line of centres is parallel to the surface

Question	Scheme	Marks	AOs
4	Use of Impulse = change in momentum	M1	3.1a
	$0.5(\mathbf{v}-8\mathbf{i})=\lambda(-\mathbf{i}+\mathbf{j})$ $(\mathbf{v}=(-2\lambda+8)\mathbf{i}+2\lambda\mathbf{j})$	A1	1.1b
	Use of Pythagoras:	M1	3.1a
	e.g. $160=(-2\lambda+8)^2+(2\lambda)^2$ $(160=4\lambda^2-32\lambda+64+4\lambda^2)$	A1	1.1b
	Form and solve quadratic in λ : $8\lambda^2-32\lambda-96=0$ $(\lambda^2-4\lambda-12=(\lambda-6)(\lambda+2)=0)$	M1	2.1
	$\Rightarrow \lambda=6$	A1	1.1b
	Find the required angle: $180°-\tan^{-1}3$	M1	1.1b
	$\theta=108$	A1	2.2a
		(8)	
			(8 marks)

Notes:

M1	Must be subtracting two values for momentum, but condone subtraction in the wrong order
A1	Correct unsimplified equation
M1	Correct use of final speed with their **v**
A1	Correct unsimplified equation in one unknown or pair of simultaneous equations
M1	Simplify and solve for λ from correct working
A1	Correct positive solution only
M1	Complete method to solve for θ
A1	108 or better (108.4349….)

Question	Scheme	Marks	AOs
5(a)	Use the model to find components of velocity after first impact:  	B1 B1	1.1b 3.4
	Kinetic energy: $\frac{1}{2}\times\frac{1}{2}mv^2=\frac{1}{2}m\left(v^2\cos^2\theta+\frac{1}{9}v^2\sin^2\theta\right)$	M1	3.1b
	$\frac{1}{2}=\frac{1}{9}+\frac{8}{9}\cos^2\theta$	M1	1.1b
	$\frac{7}{16}=\cos^2\theta,\quad \cos\theta=\frac{\sqrt{7}}{4}$	A1	1.1b
		(5)	
(a) alt	Working with initial velocity $\mathbf{v}=x\mathbf{i}-y\mathbf{j}$, after impact $\mathbf{v}=x\mathbf{i}+\frac{1}{3}y\mathbf{j}$	B1 B1	1.1b 3.4
	KE: $\frac{1}{2}\times\frac{1}{2}m\left(x^2+y^2\right)=\frac{1}{2}m\left(x^2+\frac{1}{9}y^2\right)$	M1	3.1b
	$y^2=\frac{9}{7}x^2,\quad \frac{y}{x}=\tan\theta=\frac{3}{\sqrt{7}}$	M1	1.1b
	$\cos\theta=\frac{\sqrt{7}}{4}$	A1	1.1b
		(5)	
(b)	Use the model to find components of velocity after second impact:  	B1 B1	1.1b 3.4
	Kinetic energy: $\frac{1}{4}\times\frac{1}{2}mv^2=\frac{1}{2}m\left(e^2v^2\cos^2\theta+\frac{1}{9}v^2\sin^2\theta\right)$ or $\frac{1}{2}\times\frac{1}{2}m\left(v^2\cos^2\theta+\frac{1}{9}v^2\sin^2\theta\right)=\frac{1}{2}m\left(e^2v^2\cos^2\theta+\frac{1}{9}v^2\sin^2\theta\right)$	M1	3.1b
	$\frac{1}{4}=\frac{7}{16}e^2+\frac{1}{9}\times\frac{9}{16}$	M1	1.1b
	$\Rightarrow e^2=\frac{3}{7},\quad e=\sqrt{\frac{3}{7}}$	A1	1.1b
		(5)	
(b) alt	After second impact $\mathbf{v}=-ex\mathbf{i}+\frac{1}{3}y\mathbf{j}$	B1 B1	1.1b 3.4

	KE: $\frac{1}{4}\times\frac{1}{2}m\left(x^2+y^2\right)=\frac{1}{2}m\left(e^2x^2+\frac{1}{9}y^2\right)$	M1	3.1b
	$4e^2x^2+\frac{4}{9}y^2=x^2+y^2,\quad 4e^2=1+\frac{5}{9}\left(\frac{y}{x}\right)^2$	M1	1.1b
	$\Rightarrow e^2=\frac{3}{7},\quad e=\sqrt{\frac{3}{7}}$	A1	1.1b
		(5)	
			(10 marks)

Notes:	
(a) B1 **B1**	Parallel component correct Perpendicular component correct Check the diagram
M1	Equation for KE in v, θ . Dimensionally correct. Includes all components. Condone $\frac{1}{2}$ used on wrong side
M1	Form and solve equation in $\cos\theta$
A1	Or exact equivalent
(b) B1 **B1**	Parallel component correct Perpendicular component correct
M1	Equation for KE in x,y. Dimensionally correct. Includes all components. Condone $\frac{1}{2}$ used on wrong side
M1	Use their $\cos\theta$ to form and solve equation in e
A1	Or exact equivalent

Question	Scheme	Marks	AOs
6(a)	Thrust in the spring $= \frac{3mg\,2l}{3l} \quad (= 2mg)$	B1	2.1
	Equation of motion:	M1	3.3
	$2mg - mg\sin\alpha - \frac{1}{3}mg\cos\alpha = ma$ $\left(2mg - \frac{3mg}{5} - \frac{4mg}{15} = ma\right)$	A1ft A1ft	1.1b 1.1b
	$a = \frac{17g}{15}$ *	A1*	2.2a
		(5)	
(b)	Initial EPE $= \frac{3mg\,4l^2}{2\times 3l} \quad (= 2mgl)$	B1	3.4
	Gain in GPE = $mg\,2l\sin\alpha \quad \left(= \frac{6}{5}mgl\right)$	B1	3.4
	Work done against friction = $\frac{1}{3}mg\cos\alpha \times 2l \quad \left(= \frac{8}{15}mgl\right)$	B1	3.4
	Work-energy equation:	M1	3.1a
	$\frac{1}{2}mv^2 + \frac{2}{3}mgl\cos\alpha + 2mgl\sin\alpha = 2mgl$	A1	1.1b
	$v = \sqrt{\frac{8gl}{15}}$	A1	1.1b
		(6)	
			(11 marks)

Notes:

(a) B1	Correct unsimplified expression for the thrust
M1	Equation of motion. All required terms and no extras. Dimensionally correct. Condone sign errors and sin/cos confusion
A1ft **A1ft**	Unsimplified equation with at most one error (in T or their T) Correct unsimplified equation (in T or their T)
A1*	Obtain given result from correct working
(b) B1 **B1** **B1**	Use model to obtain one correct term Use model to obtain two correct terms Use model to obtain three correct terms

M1	All required terms and no extras. Dimensionally correct. Condone sign errors and sin/cos confusion.
A1	Correct unsimplified equation
A1	Accept $0.73\sqrt{gl}$

Question	Scheme	Marks	AOs
7(a)	Component parallel to the wall: $\left[\frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j}).(8\mathbf{i}+2\mathbf{j})\right]$	M1	2.1
	$=5\sqrt{2}$	A1	1.1b
	Use of impact law perpendicular to wall:	M1	3.4
	Component perpendicular to wall after impact $\frac{1}{3}\left[\frac{1}{\sqrt{2}}(-\mathbf{i}+\mathbf{j}).(8\mathbf{i}+2\mathbf{j})\right]=-\sqrt{2}$	A1	1.1b
	For a complete method to find **v**	M1	1.1b
	$\Rightarrow \mathbf{v}=(5\mathbf{i}+5\mathbf{j})+(-\mathbf{i}+\mathbf{j})=(4\mathbf{i}+6\mathbf{j})$ *	A1*	2.2a
		(6)	
(a) alt	If $\mathbf{v}=a\mathbf{i}+b\mathbf{j}$ component parallel to the wall:	M1	2.1
	$(8\mathbf{i}+2\mathbf{j}).(\mathbf{i}+\mathbf{j})=(a\mathbf{i}+b\mathbf{j}).(\mathbf{i}+\mathbf{j})$ $(a+b=10)$	A1	1.1b
	Use of impact law:	M1	3.4
	$-\frac{1}{3}(8\mathbf{i}+2\mathbf{j}).(-\mathbf{i}+\mathbf{j})=(a\mathbf{i}+b\mathbf{j}).(-\mathbf{i}+\mathbf{j})$ $(2=-a+b)$	A1	1.1b
	For a complete method to find **v**	M1	1.1b
	$\Rightarrow \mathbf{v}=(4\mathbf{i}+6\mathbf{j})$ *	A1*	2.2a
		(6)	
(a) alt 2	Angle to wall $=31°$, component parallel to the wall:	M1	2.1
	$=\sqrt{68}\cos 31°=7.07$	A1	1.1b
	Component perpendicular to the wall	M1	3.4
	$=\frac{1}{3}\sqrt{68}\sin 31°=1.42$	A1	1.1b
	For a complete method to find **v**	M1	1.1b
	$\Rightarrow \mathbf{v}=\left(\sqrt{52}\cos 56.3°\mathbf{i}+\sqrt{52}\sin 56.3°\mathbf{j}\right)=(4\mathbf{i}+6\mathbf{j})$	A1*	2.2a
		(6)	
(b)	$\mathbf{I}=0.25(4\mathbf{i}+6\mathbf{j})-0.25(8\mathbf{i}+2\mathbf{j})$ $(\mathbf{I}=0.25(-\mathbf{i}+\mathbf{j})-0.25(3\mathbf{i}-3\mathbf{j}))$ $(\mathbf{I}=(-\mathbf{i}+\mathbf{j}))$	M1	3.1b

	Use of Pythagoras	M1	1.1b
	$\lvert\mathbf{I}\rvert = \sqrt{2}\,(\text{Ns})$	A1	1.1b
		(3)	
			(9 marks)

Notes:	
(a)M1	Use of scalar product or equivalent. Allow M1 if not using unit vector
A1	Correct unsimplified expression for component parallel to wall
M1	Correct use of impact law perpendicular to the wall. Condone sign error
A1	Correct unsimplified expression for component perpendicular to wall
M1	Complete method to solve for **v**
A1*	Obtain given result from correct working
(b) M1	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ with velocities or perpendicular components of velocities. Must be subtracting but allow subtraction in either order.
M1	Correct use of Pythagoras to find modulus
A1	Accept 1.4 Ns or better