Level 2 Certificate
FURTHER MATHEMATICS
8365/1

Paper 1 Non-Calculator

Mark scheme

June 2022

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
3.14…	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416

Examiners should consistently apply the following principles.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

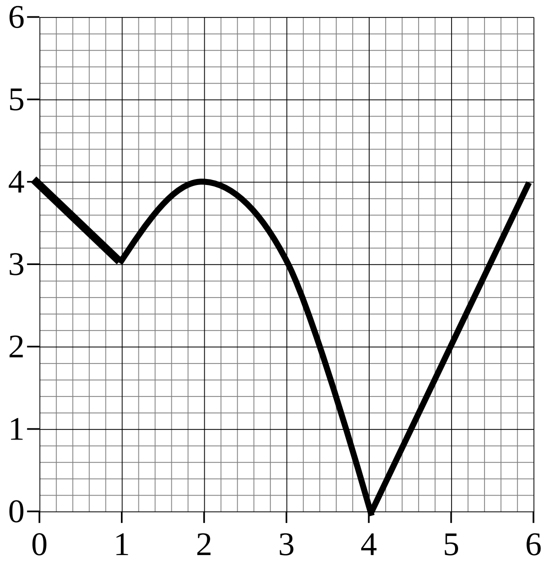
Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Continental notation

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

<table>
<tr><th>Question</th><th>Answer</th><th>Mark</th><th>Comments</th></tr>
<tr><td rowspan="10">1</td><td colspan="3">Alternative method 1</td></tr>
<tr><td>1.2 or $\frac{6}{5}$</td><td>M1</td><td>oe could be seen in calculation (120% is not M1 – something needs to have been done with it)
$\frac{5}{6}$ if used correctly could be an oe. Don’t award just for $\frac{5}{6}$ seen</td></tr>
<tr><td>$1.2x + 1.2 = x + 6$
or $0.2x + 1.2 = 6$
or $0.2x = 4.8$</td><td>M1dep</td><td>oe but must have expanded brackets
missing brackets need to be recovered</td></tr>
<tr><td>24</td><td>A1</td><td></td></tr>
<tr><td colspan="3">Alternative method 2</td></tr>
<tr><td>$(x + 1) + \frac{(x+1)}{5}$</td><td>M1</td><td>oe</td></tr>
<tr><td>$\frac{(x+1)}{5} = 5$
or $(x + 1) = 25$</td><td>M1dep</td><td>oe
eg could be written as 20% of $(x + 1) = 5$</td></tr>
<tr><td>24</td><td>A1</td><td></td></tr>
<tr><td colspan="3">Additional Guidance</td></tr>
<tr><td colspan="2">20% = 5 or 100% = 25
$1.2(x +1) = x + 6$ then $1.2x + 1 = x + 6$ would not gain second M mark</td><td>SC1</td></tr>
</table>

<table>
<tr><th>Question</th><th>Answer</th><th>Mark</th><th>Comments</th></tr>
<tr><td rowspan="12">2</td><td colspan="3">Alternative method 1</td></tr>
<tr><td>$-4 = \frac{3}{2} \times -6 + c$ or $c = 5$
$y - -4 = \frac{3}{2}(x - -6)$</td><td>M1</td><td>oe</td></tr>
<tr><td>(0, 5)</td><td>A1</td><td></td></tr>
<tr><td colspan="3">Alternative method 2</td></tr>
<tr><td>Correctly adding at least 1 multiple of 2 to the right and 3 up
eg –6 + 2 = –4 and –4 + 3 = –1</td><td>M1</td><td>oe needs to be added to both vertical and horizontal. Could be seen in coordinates eg (–4, –1)
could be 1 right and 1.5 up
or y coordinate of $-4 + 1.5 \times 6$</td></tr>
<tr><td>(0, 5)</td><td>A1</td><td></td></tr>
<tr><td colspan="3">Alternative method 3</td></tr>
<tr><td>Sketch drawn with straight line passing through (–6, –4) and (0, 5) with steps shown</td><td>M1</td><td>just a line passing through 5 seen on the axis is enough for M1 but won’t gain A1 unless written as coordinates</td></tr>
<tr><td>(0, 5)</td><td>A1</td><td>answer could be embedded in diagram</td></tr>
<tr><td colspan="3">Additional Guidance</td></tr>
<tr><td colspan="2">(0, 5) seen without working will be 2 marks</td><td>M1A1</td></tr>
</table>

<table>
<tr><th>Question</th><th>Answer</th><th>Mark</th><th>Comments</th></tr>
<tr><td rowspan="6">3 (a)</td><td>Line joining (0,4) and (1,3)</td><td>B1</td><td>may be drawn free hand</td></tr>
<tr><td>(2, 4) plotted as a maximum value</td><td>M1</td><td>needs to be some sort of graph showing a maximum value</td></tr>
<tr><td>Curve drawn through
(1, 3), (2, 4), (3, 3) and (4, 0)</td><td>A1</td><td>all points should be within half a square horizontally or vertically</td></tr>
<tr><td>Line joining (4,0) and (6,4)</td><td>B1</td><td>may be drawn free hand</td></tr>
<tr><td colspan="3">Additional Guidance</td></tr>
<tr><td colspan="2">
Maximum mark available if either or both of the straight lines include a curve (they haven’t used a ruler) is B1M1A1
Maximum mark available if any part of the quadratic curve (it must be a quadratic curve that is concave and not convex at any point) is drawn with a ruler is B2M1 (a clear vertex at (3,3) may show the use of a ruler)
Maximum mark available if any of the lines go beyond their correct domain by more than half a square is B2M1 or B1M1A1
They could lose marks for both the quadratic and straight lines
Ignore slight feathering</td><td></td></tr>
</table>

Question	Answer	Mark	Comments
3 (b)	**Alternative method 1**		
	Rearranging first to get $x = \frac{6 - g(x)}{3}$	M1	oe eg $x = \frac{6-y}{3}$ or $2 - \frac{y}{3}$ $y - 6 = -3x$ is not enough to gain M1
	$g^{-1}(x) = \frac{6-x}{3}$	A1	oe eg $g^{-1}(x) = \frac{x-6}{-3}$ or $g^{-1}(x) = -\frac{x-6}{3}$ or $g^{-1}(x) = \frac{x}{-3} + 2$
	Alternative method 2		
	Putting the correct terminology in to get $x = 6 - 3g^{-1}(x)$	M1	oe eg $x = 6 - 3y$ or $3y = 6 - x$
	$g^{-1}(x) = \frac{6-x}{3}$	A1	oe eg $g^{-1}(x) = \frac{x-6}{-3}$ or $g^{-1}(x) = -\frac{x-6}{3}$ or $g^{-1}(x) = \frac{x}{-3} + 2$
	Additional Guidance		
	Answer left as $y = \frac{6-x}{3}$ should gain M1 on either scheme		M1A0
	$x = \frac{6-y}{3}$ can gain M1 but not A1		M1A0
	Condone $g^{-1}(x)$ missed on answer line (as long as nothing else is written in its place)		
	Flow charts may be used. Mark as oe		
	Penalise additional incorrect working		

Question	Answer	Mark	Comments
4 (a)	$\frac{1}{3}$	B1	
	Additional Guidance		

Question	Answer	Mark	Comments
4 (b)	Any line through (0, 1), (90, 0), (180, –1), (270, 0) and (360, 1)	M1	± 2mm either side for the points on the axes but for (180, –1) and (360, 1) mark intention
	Correct graph drawn	A1	mark intention
	Additional Guidance		
	Ignore slight feathering. Lines should be curves (any sight of a ruler being used will lose A1) Ignore curve going beyond 0 or 360		

Question	Answer	Mark	Comments
5	$15x^2 - 12x + 5ax - 4a$ or $5ax - 12x = -2x$ or $5a - 12 = -2$ or $b = -4a$	M1	oe
	$(a =)\ 2$	A1	
	$(b =)\ -4 \times$ their a	A1ft	-8, but do not award -8 unless it comes from $a = 2$
	Additional Guidance		
	Candidates who use substitutions for x are likely to use $x = 0$ and gain M1. Award M1 for any number substituted in correctly to gain an equation in a and b		

Question	Answer	Mark	Comments
6	**Alternative method 1**		
	$(y =)\ 2x^7 + 4x^4 - 6x^3$	M1	for any 2 terms correct
	$\left(\frac{dy}{dx} =\right)\ 14x^6 + 16x^3 - 18x^2$	A2	oe eg $2x^2(7x^4 + 8x - 9)$ A1 for any correct term correctly differentiated
	Alternative method 2 (product rule)		
	$\left(\frac{dy}{dx} =\right)$ $8x^3(x^3 + 2 - \frac{3}{x}) + 2x^4(3x^2 + 3x^{-2})$	M1	for either $2x^4$ differentiated correctly multiplied by the bracket or the bracket differentiated correctly multiplied by $2x^4$ eg $8x^3(x^3 + 2 - \frac{3}{x})$
	$\left(\frac{dy}{dx} =\right)\ 14x^6 + 16x^3 - 18x^2$	A2	oe eg $2x^2(7x^4 + 8x - 9)$ A1 for any term correct
	Additional Guidance		
	Ignore subsequent incorrect factorisation Condone incorrect use of $y =$ on the answer line		

Question	Answer	Mark	Comments
7	(AB =) 1 and (AC =) 0.75	M1	oe could be seen on diagram allow AB = –1 and/or AC = –0.75
	$(BC^2 =)\ 1^2 + \left(\frac{3}{4}\right)^2$	M1dep	oe eg $(-2 - -1)^2 + \left(5\frac{3}{4} - 5\right)^2$ $\sqrt{1.5625}$ or $\sqrt{\frac{25}{16}}$ or $\sqrt{1\frac{9}{16}}$ would imply this mark
	(BC =) $\frac{5}{4}$ or $1\frac{1}{4}$ or 1.25	A1	
	Additional Guidance		
	Candidates may spot it's a $\frac{3}{4}$,1, $\frac{5}{4}$ Pythagorean triple which gains the M marks and will probably go on to score all marks Ignore further rounding or truncating after correct answer seen eg $\frac{5}{4}$ followed by = 1.2 would score the A mark Condone $\frac{3}{4}^2$ without the brackets. Condone -1^2 without the brackets		M2A1
	$\frac{5}{4}$ followed by = 0.8 is incorrect further working		M2A0

Question	Answer	Mark	Comments
8	(Reflection =) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ or (Rotation =) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	M1	no words needed but if they are labelled incorrectly then M0 correct matrices could be seen amongst many others
	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	M1dep	both matrices correct and written in correct order
	$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	A1	matrices multiplied out correctly this matrix should be at the end of the proof and should not be amongst other matrices
	Additional Guidance		
	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$		SC2
	Condone matrix written without brackets for M marks		M2A0
	Correct multiplication of a unit square by both matrices will imply both M marks but won't get the A mark unless the reflection matrix is shown. The matrix multiplications would need to be done in the correct order however		M2A0
	Trying to operate matrices on a single point will only gain the first M mark (as it wouldn't necessarily be true for all points). It would still require a correct matrix though		M1A0
	Some candidates may multiply matrices in a grid. If $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is the result then the M1dep mark can be awarded. If not, do not award M1dep		
	Condone commas in matrices for M marks but will lose the A mark		M2A0

Question	Answer	Mark	Comments
9(a)	**Alternative method 1 (grid)**		
	1 5 9 4 4 and $2n^2$	M1	
	–4 –9 (–14 –19) and –5n (+ c)	M1dep	subtract $2n^2$
	$2n^2 - 5n + 1$	A1	
	Alternative method 2 (simultaneous equations)		
	Any 3 of: $a + b + c = -2$ $4a + 2b + c = -1$ $9a + 3b + c = 4$ $16a + 9b + c = 13$	M1	using n^{th} term = $an^2 + bn + c$
	$3a + b = 1$ or $5a + b = 5$	M1dep	or any other equation with an unknown eliminated
	$b = -5$, $c = 1$ so $2n^2 - 5n + 1$	A1	
	Alternative method 3 (using terms)		
	1 5 9 4 4 so $a = 2$	M1	using n^{th} term = $an^2 + bn + c$
	$3a + b = 1$ and $a = 2$ substituted in this equation	M1dep	oe
	$b = -5$, $c = 1$ so $2n^2 - 5n + 1$	A1	
	Additional Guidance		
	Condone other letters used eg $2x^2 - 5x + 1$ or even $2n^2 - 5x + 1$		
	After finding $a = 2$ they may find the 0th term to get $c = 1$	M2	
	$2n^2 + 5n - 1$ from Alt 1 but subtracting the wrong way round	SC2	

Question	Answer	Mark	Comments
9(b)	$n^2 + 10n - 2000 < 0$	M1	the correct inequality needed for this mark and must be written in this form
	$(n - 40)(n + 50)$ or $(n + 5)^2 - 25 - 2000$ or $\dfrac{-10 \pm \sqrt{10^2 - 4 \times 1 \times -2000}}{2}$	M1	oe inequality not needed for this mark condone + instead of $\pm$ as the negative solution has no meaning here
	39	A1	
	Additional Guidance		
	Do not accept T&I		
	Incorrect use of inequalities can be recovered by a correct use of inequalities later in the method such as $n < 40$ near the end	M2	
	Incorrect use of inequalities can be recovered for full marks. An answer of 39 after a method that uses an incorrect inequality or = shows inequality has been recovered	M2A1	
	An incorrect solution with incorrect use of inequalities can only be awarded the second M mark	M0M1A0	
	Correct answer not coming from correct working will not gain any marks	M0A0	
	For students who try to complete the square accept $(n + 5)^2 < 2025$ as an oe giving M2 but $(n + 5)^2 = 2025$ would only gain M0M1 unless recovered in the answer		

*

Question	Answer	Mark	Comments
10	**Alternative method 1**		
	$\dfrac{\sqrt{3}}{3+\sqrt{3}} \times \dfrac{3-\sqrt{3}}{3-\sqrt{3}}$ or $\dfrac{\sqrt{3}}{3+\sqrt{3}} \times \dfrac{\sqrt{3}-3}{\sqrt{3}-3}$	M1	$\times (3 - \sqrt{3})$ can still gain full marks if recovered but doesn't gain M1 if the second M mark isn't awarded
	$\dfrac{3\sqrt{3}-3}{9-3}$	M1dep	oe eg $\dfrac{3\sqrt{3}-\sqrt{3}\sqrt{3}}{9+3\sqrt{3}-3\sqrt{3}-\sqrt{3}\sqrt{3}}$ or $\dfrac{3\sqrt{3}-\sqrt{9}}{9+3\sqrt{3}-3\sqrt{3}-\sqrt{9}}$ or $\dfrac{3\sqrt{3}}{6}-\dfrac{3}{6}$
	$\dfrac{\sqrt{3}-1}{2}$	A1	oe to something fully simplified eg $\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}$ or $\dfrac{1-\sqrt{3}}{-2}$
	Alternative method 2		
	$\dfrac{\sqrt{3}}{3+\sqrt{3}} = \dfrac{1}{\sqrt{3}+1}$	M1	
	$\dfrac{1}{\sqrt{3}+1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$ or $\dfrac{1}{\sqrt{3}+1} \times \dfrac{1-\sqrt{3}}{1-\sqrt{3}}$	M1dep	oe $\times (\sqrt{3} - 1)$ can still gain full marks if recovered but doesn't gain M1 if the A mark isn't awarded
	$\dfrac{\sqrt{3}-1}{2}$	A1	oe eg $\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}$ or $\dfrac{1-\sqrt{3}}{-2}$

	Alternative method 3		
	$\dfrac{\sqrt{3}}{3+\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{3}{3\sqrt{3}+3}$	M1	
	$\dfrac{3}{3\sqrt{3}+3} \times \dfrac{3\sqrt{3}-3}{3\sqrt{3}-3}$ or $\dfrac{3}{3\sqrt{3}+3} \times \dfrac{3-3\sqrt{3}}{3-3\sqrt{3}}$	M1dep	oe × (3$\sqrt{3}$ – 3) can still gain full marks if recovered but doesn't gain M1 if the A mark isn't awarded
	$\dfrac{\sqrt{3}-1}{2}$	A1	oe eg $\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}$ or $\dfrac{1-\sqrt{3}}{-2}$
	Additional Guidance		
	Penalise further incorrect working		

Question	Answer	Mark	Comments
11	**Alternative method 1**		
	Evidence of 1 5 10 10 5 1 **used** for all six coefficients (terms could be written incorrectly)	M1	the 1s can be ignored but 5 10 10 5 must be seen and **used** (don't accept it just being written in Pascal's triangle)
	$(3)^5 + 5(3)^4(2x) + 10(3)^3(2x)^2 + 10(3)^2(2x)^3 + 5(3)(2x)^4 + (2x)^5$	M1dep	oe eg $(3)^5(2x)^0$ written for first term at least 4 terms correct (could already be simplified and missing brackets recovered)
	$(3)^5 + 5(3)^4(2x) + 10(3)^3(2x)^2 + 10(3)^2(2x)^3 + 5(3)(2x)^4 + (2x)^5$	M1dep	oe eg $(3)^5(2x)^0$ written for first term all correct
	$243 + 810x + 1080x^2 + 720x^3 + 240x^4 + 32x^5$	A1	
	Alternative method 2		
	$(3 + 2x)^2 = 9 + 12x + 4x^2$	M1	
	$(3 + 2x)^3 = 27 + 54x + 36x^2 + 8x^3$	M1dep	oe the terms may not have been collected could do $(3 + 2x)^2 \times (3 + 2x)^2$. If they use this method (doesn't refer to $(3 + 2x)^3$) then award this mark for answer expanded correctly but with one numerical error. Terms must be collected
	$(3 + 2x)^4 = 81 + 216x + 216x^2 + 96x^3 + 16x^4$	M1dep	terms must be collected could do $(3 + 2x)^2 \times (3 + 2x)^3$. If they use this method (doesn't refer to $(3 + 2x)^4$) then award this mark for answer expanded correctly but with one numerical error. Terms must be collected would imply first 2 M marks if done correctly
	$243 + 810x + 1080x^2 + 720x^3 + 240x^4 + 32x^5$	A1	

	Alternative method 3		
	Evidence of 1 5 10 10 5 1 used for all six coefficients (could be written incorrectly)	M1	the 1s can be ignored but 5 10 10 5 must be seen and used
	$a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$	M1dep	from using a general expansion of $(a + b)^5$
	$(3)^5 + 5(3)^4(2x) + 10(3)^3(2x)^2 + 10(3)^2(2x)^3 + 5(3)(2x)^4 + (2x)^5$	M1dep	oe all correct
	$243 + 810x + 1080x^2 + 720x^3 + 240x^4 + 32x^5$	A1	
	Additional Guidance		
	Working could be seen as a list or a grid. This can be awarded full marks if done correctly Candidates could use a combination of methods. Use whichever alt method works best (probably alt 2) Missing brackets must be recovered		M3A1

Question	Answer	Mark	Comments
12(a)	$33n^2 = 32(n^2 + 2)$ or $\dfrac{64 - n^2}{11n^2 + 22} = 0$	M1	oe (both denominators should be cleared for the first method)
	8	A1	ignore –8 in working as long as only 8 stated in answer
	Additional Guidance		
	May use T&I and will be 2 marks if they get the correct answer (0 marks without the answer)		

Question	Answer	Mark	Comments
12(b)	3	B1	
	Additional Guidance		
	Condone $\frac{3}{1}$		

Question	Answer	Mark	Comments
13	2 and 3	B1	coefficients
	x and x^3	B1	
	y and y^4	B1	
	Additional Guidance		
	$2xy + 3x^3y^4$ or $xy(2 + 3x^2y^3)$ scores B3		B3
	If no B marks awarded then $3x^3(2y^{-2} + 3x^2y)$ or $3x^3y(2y^{-3} + 3x^2)$ or $3x^3y^{-2}(2 + 3x^2y^3)$ or $3x^2(2xy^{-2} + 3x^3y)$ or $3x^2y(2xy^{-3} + 3x^3)$ or $3x^2y^{-2}(2x + 3x^3y^3)$ seen in the working for the numerator		SC1
	Penalise incorrect further working for the B marks		

Question	Answer	Mark	Comments
14	**Alternative method 1**		
	$3ef = 5e + 4$ or $ef - \dfrac{5e}{3} = \dfrac{4}{3}$	M1	
	$e(3f - 5) = 4$ or $e\left(f - \dfrac{5}{3}\right) = \dfrac{4}{3}$	M1dep	oe where they are one step away from answer
	$e = \dfrac{4}{3f - 5}$	A1	oe eg $e = \dfrac{\frac{4}{3}}{\left(f - \frac{5}{3}\right)}$ or $e = \dfrac{-4}{5 - 3f}$
	Alternative method 2		
	$3f = 5 + \dfrac{4}{e}$	M1	
	$\dfrac{4}{e} = 3f - 5$	M1dep	oe where they are one step away from answer
	$e = \dfrac{4}{3f - 5}$	A1	oe eg $e = \dfrac{\frac{4}{3}}{\left(f - \frac{5}{3}\right)}$ or $e = \dfrac{-4}{5 - 3f}$
	Additional Guidance		
	Must have $e =$ on the answer line for full marks		

Question	Answer	Mark	Comments
15	States that $\angle$ABP or $\angle$ACP is 90	B1	can be seen on diagram (either 90 or a square angle)
	Any one further angle correct (not $\angle$ABP or $\angle$ACP)	B1	minor $\angle$BPC = 180 – x or 360 – 2y or major $\angle$BPC = 2y or 180 + x or $\angle$BQC = 180 – y or 90 – $\frac{x}{2}$ (where Q is a point on the major arc)
	Another further angle correct (not $\angle$ABP or $\angle$ACP)	B1	any two of minor $\angle$BPC = 180 – x or 360 – 2y or major $\angle$BPC = 2y or 180 + x or $\angle$BQC = 180 – y or 90 – $\frac{x}{2}$ (where Q is a point on the major arc) could be the same angle found in the previous B mark but an expression in y rather than x
	A correct equation in terms of x and y and rearrange to $y = 90 + \frac{x}{2}$	B1dep	dependent on first three B marks awarded doesn't imply the first 3 B marks
	3 reasons given for the theorems used correctly for the angles stated in the first three marks	B1dep	dependent on first three B marks awarded reason - angle formed from a tangent and a radius is a right angle (can only be used once) reason - angles in a quadrilateral add up to 360 reason - angle at the centre is twice the angle at the circumference reason - opposite angles in a cyclic quadrilateral add up to 180 reason - angles at a point (or in a circle) add up to 360 reason - alternate segment theorem

	Additional Guidance	
	Angles must be identified with either our terminology such as $\angle$ABP or their own labelling such as m or θ or can be seen on the diagram Accept supplementary for angles adding to 180 Accept complementary for angles adding to 90 Use of obtuse and reflex or interior and exterior instead of minor and major is fine. If it's not clear then assume it's the minor arc they are referring to Check candidates are not assuming that BDCP is a kite and using symmetry of this shape Check candidates are not using BDCP as a cyclic quadrilateral No credit for numbers used instead of x and y Mark the first three B marks positively Note – ABPC is a cyclic quadrilateral but D is not the centre of that circle Note – D is not the middle of minor arc BC	

Question	Answer	Mark	Comments
16	**Alternative method 1**		
	Correct substitution $x - \frac{-3}{x} = \frac{19}{4}$ or $x\left(x - \frac{19}{4}\right) = -3$	M1	penalise no brackets unless recovered
	$4x^2 - 19x + 12$ (= 0)	M1dep	oe eg $4x^2 + 12 = 19x$ must be integer values unless going on to complete the square
	$(4x + a)(x + b)$ or $(4x - 3)(4x - 16)$	M1dep	where $ab = 12$ or $a + 4b = -19$
	$(4x - 3)(x - 4)$	A1	
	$x = \frac{3}{4}$ and 4 or $x = \frac{3}{4}$ and $y = -4$ or $x = 4$ and $y = -\frac{3}{4}$	A1	
	$y = -4$ and $-\frac{3}{4}$ or $x = 4$ and $y = -\frac{3}{4}$ or $x = \frac{3}{4}$ and $y = -4$	A1	all 4 values must be correct to gain this mark
	Alternative method 2		
	Correct substitution $x - \frac{-3}{x} = \frac{19}{4}$ or $x\left(x - \frac{19}{4}\right) = -3$	M1	penalise no brackets unless recovered
	$4x^2 - 19x + 12$ (= 0)	M1dep	oe eg $4x^2 + 12 = 19x$ must be integer values unless going on to complete the square

	$\dfrac{19 \pm \sqrt{19^2 - 4 \times 4 \times 12}}{2 \times 4}$	M1dep	
	$\dfrac{19 \pm \sqrt{169}}{8}$	A1	
	$x = \dfrac{3}{4}$ and 4 or $x = \dfrac{3}{4}$ and $y = –4$ or $x = 4$ and $y = -\dfrac{3}{4}$	A1	
	$y = –4$ and $-\dfrac{3}{4}$ or $x = 4$ and $y = -\dfrac{3}{4}$ or $x = \dfrac{3}{4}$ and $y = –4$	A1	all 4 values must be correct to gain this mark
	Alternative method 3		
	Correct substitution $x - \dfrac{-3}{x} = \dfrac{19}{4}$ or $x\left(x - \dfrac{19}{4}\right) = –3$	M1	penalise no brackets unless recovered
	$4x^2 – 19x + 12$ (= 0) or $x^2 - \dfrac{19}{4}x + 3$ (= 0)	M1dep	oe eg $4x^2 + 12 = 19x$ must be integer values unless going on to complete the square
	$4\left[\left(x - \dfrac{19}{8}\right)^2 .....\right] .....$ or $\left[\left(x - \dfrac{19}{8}\right)^2 .....\right] .....$	M1	oe

	$4\left(x-\frac{19}{8}\right)^2-\frac{169}{16}=0$ or $\left[\left(x-\frac{19}{8}\right)^2\right]-\frac{169}{64}=0$	M1dep	
	$x=\frac{3}{4}$ and 4 or $x=\frac{3}{4}$ and $y=-4$ or $x=4$ and $y=-\frac{3}{4}$	A1	
	$y=-4$ and $-\frac{3}{4}$ or $x=4$ and $y=-\frac{3}{4}$ or $x=\frac{3}{4}$ and $y=-4$	A1	all 4 values must be correct to gain this mark
	Alternative method 4		
	Correct substitution $\frac{-3}{y}-y=\frac{19}{4}$ or $y\left(y+\frac{19}{4}\right)=-3$	M1	penalise no brackets unless recovered
	$4y^2+19y+12$ (= 0)	M1dep	oe eg $4y^2+12=-19y$ must be integer values unless going on to complete the square
	$(4y+a)(y+b)$ or $(4y+3)(4y+16)$	M1dep	where $ab=12$ or $a+4b=19$
	$(4y+3)(y+4)$	A1	
	$y=-\frac{3}{4}$ and -4 or $y=-\frac{3}{4}$ and $x=4$ or $y=-4$ and $x=\frac{3}{4}$	A1	

	$x = 4$ and $\frac{3}{4}$ or $y = -\frac{3}{4}$ and $x = 4$ or $y = -4$ and $x = \frac{3}{4}$	A1	all 4 values must be correct to gain this mark
	Alternative method 5		
	Correct substitution $\frac{-3}{y} - y = \frac{19}{4}$ or $y\left(y + \frac{19}{4}\right) = -3$	M1	penalise no brackets unless recovered
	$4y^2 + 19y + 12$ (= 0)	M1dep	oe eg $4y^2 + 12 = -19y$ must be integer values unless going on to complete the square
	$\frac{-19 \pm \sqrt{19^2 - 4 \times 4 \times 12}}{2 \times 4}$	M1dep	
	$-\frac{19 \pm \sqrt{169}}{8}$	A1	
	$y = -\frac{3}{4}$ and -4 or $y = -\frac{3}{4}$ and $x = 4$ or $y = -4$ and $x = \frac{3}{4}$	A1	
	$x = 4$ and $\frac{3}{4}$ or $y = -\frac{3}{4}$ and $x = 4$ or $y = -4$ and $x = \frac{3}{4}$	A1	all 4 values must be correct to gain this mark

	Alternative method 6		
	Correct substitution $\frac{-3}{y} - y = \frac{19}{4}$ or $y\left(y + \frac{19}{4}\right) = -3$	M1	penalise no brackets unless recovered
	$4y^2 + 19y + 12$ (= 0) or $y^2 + \frac{19}{4}y + 3$ (= 0)	M1dep	oe eg $4y^2 + 12 = -19y$ must be integer values unless going on to complete the square
	$4\left[\left(y + \frac{19}{8}\right)^2 .....\right] .....$ or $\left[\left(y + \frac{19}{8}\right)^2 .....\right] .....$	M1	oe
	$4\left(y + \frac{19}{8}\right)^2 - \frac{169}{16} = 0$ or $\left[\left(y + \frac{19}{8}\right)^2\right] - \frac{169}{64} = 0$	M1dep	
	$y = -\frac{3}{4}$ and –4 or $y = -\frac{3}{4}$ and $x = 4$ or $y = -4$ and $x = \frac{3}{4}$	A1	
	$x = 4$ and $\frac{3}{4}$ or $y = -\frac{3}{4}$ and $x = 4$ or $y = -4$ and $x = \frac{3}{4}$	A1	all 4 values must be correct to gain this mark
	Additional Guidance		
	Correct A marks must come from correct algebra in M marks		

Question	Answer	Mark	Comments
17(a)	**Alternative method 1**		
	Radius of circle = 4	M1	4 could be seen in the solution or diagram without the word radius stated
	Use of 4cos 60 and 4sin 60 and $4 \times \frac{1}{2}$ and $4 \times \frac{\sqrt{3}}{2}$	A1	$= (2, 2\sqrt{3})$ candidates could use the sine rule but it should look like this anyway
	Alternative method 2		
	$1 : \sqrt{3} : 2$ triangle seen or stated	M1	Pythagorean triple
	$2 : 2\sqrt{3} : 4$	A1	
	Alternative method 3		
	$\tan 60 = \frac{\text{opp}}{\text{adj}} = \frac{2\sqrt{3}}{2} = \sqrt{3}$ or $\frac{\text{opp}}{\text{adj}} = \frac{2\sqrt{3}}{2} = \sqrt{3} = \tan 60$	B1	shows that the point is on the line OP
	$(2\sqrt{3})^2 + 2^2 = 12 + 4 = 16$	B1	shows that the point lies on the circle
	Additional Guidance		
	Candidates could find one coordinate and then substitute into the circle equation to show the second coordinate		M1A1
	Candidates may try to use multiple alt methods – mark according to the method that gives them the best mark		
	It is possible to show that the x coordinate is 2 by connecting P and (4,0) hence creating an equilateral triangle (this would need to be stated). Then drop a perpendicular from P which bisects the base line showing that the x coordinate is 2		

Question	Answer	Mark	Comments
17(b)	(Gradient of OP =) $\frac{2\sqrt{3}}{2}$ or = $\sqrt{3}$	M1	$\sqrt{3}$ either from part (a) or knowing that an angle of 60° gives it
	(Gradient of tangent =) $\frac{-1}{\text{their}\sqrt{3}}$	M1	oe $\frac{-1}{\sqrt{3}}$ would imply the first M mark
	$y - 2\sqrt{3} = \frac{-1}{\sqrt{3}}(x - 2)$ or $2\sqrt{3} = \frac{-1}{\sqrt{3}}(2) + c$	M1dep	oe dependent on M2 already being awarded $c = \frac{8\sqrt{3}}{3}$
	$x + \sqrt{3}\,y = 8$	A1	
	Additional Guidance		

Question	Answer	Mark	Comments
18	$\sin 135° = \frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$	B1	oe could be embedded in the calculation
	$\frac{\sin\theta}{x} = \frac{\sin 135}{4x}$	M1	oe x may be a different letter (would need to be the same letter for both sides) or numerical values using multiples of 1 and 4 used
	$(\sin\theta =)\frac{\sqrt{2}}{8}$	A1	oe eg $\frac{1}{4\sqrt{2}}$
	Additional Guidance		
	Penalise incorrect further working Condone $\theta = \sin^{-1}\frac{\sqrt{2}}{8}$ seen in answer as long as $\sin\theta = \frac{\sqrt{2}}{8}$ seen in working		

Question	Answer	Mark	Comments
19	**Alternative method 1**		
	$6(x^2 - 4x$ ……. or $6(x - 2)^2$ ……….	M1	oe eg $6[(x^2 - 4x)$……]
	$6[(x - 2)^2 - 2^2]$…….. or $6[(x - 2)^2 - 4]$……. or $6[(x - 2)^2 - 4 + \frac{17}{6}]$ or $6[(x - 2)^2 - \frac{7}{6}]$ or $6(x - 2)^2 - 6 \times \frac{7}{6}$ or $6(x - 2)^2 - 24 + 17$	M1dep	oe the bracket is after the 2^2 and the 4 here. If they put something else inside the bracket it is incorrect unless it is equivalent to one of the fully complete versions listed
	$6(x - 2)^2 - 7$	A1	
	Alternative method 2		
	$ax^2 + 2abx + ab^2$ $(+c)$	M1	expansion of brackets
	$a = 6$ and $2ab = -24$ and $ab^2 + c = 17$	M1dep	
	$b = -2$ and $c = -7$	A1	
	Additional Guidance		

Question	Answer	Mark	Comments
20	$\left(\frac{dy}{dx} =\right) 4x^3 - 36x$	M1	either term correct
	their $\frac{dy}{dx} = 0$	M1dep	could be written as $x(x^2 - 9) = 0$ or $4x(x^2 - 9) = 0$ follow through an incorrect differentiation as long as it has at least one term correct
	$4x(x + 3)(x - 3)$ (= 0)	M1dep	oe eg $x(x + 3)(x - 3)$ (= 0) solutions could be gained by using the factor theorem
	(–3, –81) (0, 0) (3, –81)	A1	may be seen in calculation rather than put in coordinates at this stage
	$\left(\frac{d^2y}{dx^2} =\right) 12x^2 - 36$ and when $x = -3$ $\left(\frac{d^2y}{dx^2}\right) = 72$ and/or positive or when $x = 0$ $\left(\frac{d^2y}{dx^2}\right) = -36$ and/or negative or when $x = 3$ $\left(\frac{d^2y}{dx^2}\right) = 72$ and/or positive or any check to both sides of one of their solutions to give one side with a negative gradient and one side with a positive gradient	M1dep	dependent on M3 oe correct y coordinates not required for this M mark any one point assessed correctly (don't need to state max or min at this stage) but if the value of f''(x) is worked out incorrectly then penalise. The value of f''(x) may not be shown and then the correct statement will suffice. eg $x = -4$ $\frac{dy}{dx} < 0$ $x = -1$ $\frac{dy}{dx} > 0$ $x = 1$ $\frac{dy}{dx} < 0$ $x = 4$ $\frac{dy}{dx} > 0$

	(–3, –81) Minimum (0, 0) Maximum (3, –81) Minimum	A1	all three points must have been determined correctly to gain this mark this could imply the previous mark by use of a correct sketch graph or a statement that says a positive quartic has these stationary points
	Additional Guidance		
	Condone incorrect writing of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ even if it's just $y =$ as long as it's recovered to get the correct nature of the turning points		

Question	Answer	Mark	Comments
21	**Alternative method 1**		
	LHS Use of: $\cos^2 x \equiv 1 - \sin^2 x$ or $\sin^2 x \equiv 1 - \cos^2 x$ or $3\sin^2 x + 3\cos^2 x \equiv 3$ in numerator to get: $4(1 - \sin^2 x) + 3\sin^2 x - 4$ or $4\cos^2 x + 3(1 - \cos^2 x) - 4$ or $3 + \cos^2 x - 4$	M1	oe must be used as part of a solution (nothing for just stating it)
	LHS $\dfrac{4 - 4\sin^2 x + 3\sin^2 x - 4}{\cos^2 x}$ or simplification of the other forms leading to $\dfrac{\cos^2 x - 1}{\cos^2 x}$	M1dep	one step away from the A mark this could imply the first M1 provided they have stated the identity used from the list in the first M mark
	$-\dfrac{\sin^2 x}{\cos^2 x} \equiv -\tan^2 x$	A1	oe
	Alternative method 2		
	LHS $\dfrac{\left[4\cos^2 x + 3\sin^2 x - 4\left(\cos^2 x + \sin^2 x\right)\right]}{\cos^2 x}$	M1	
	$\dfrac{\left[4\cos^2 x + 3\sin^2 x - 4\cos^2 x - 4\sin^2 x\right]}{\cos^2 x}$	M1	
	$-\dfrac{\sin^2 x}{\cos^2 x} \equiv -\tan^2 x$	A1	oe

	Alternative method 3		
	RHS $-\tan^2 x \equiv -\dfrac{\sin^2 x}{\cos^2 x}$	M1	
	$\dfrac{\left[4\left(\sin^2 x + \cos^2 x\right) - 4 - \sin^2 x\right]}{\cos^2 x}$	M1	
	$\dfrac{\left[4\cos^2 x + 3\sin^2 x - 4\right]}{\cos^2 x}$	A1	
	Additional Guidance		
	Either starts with the left and finishes with the right or vice versa. Max M2 for any working that meets in the middle by trying to solve an equation Only mark using one of the alts – once the candidate starts to treat the solution as an equation by moving terms around from one side of the ≡ to the other then stop awarding marks The exception to this would be if a candidate uses identities to manipulate the LHS to an expression correctly and also then manipulates the RHS correctly to the same expression. They would then need to state that these two manipulations show the LHS ≡ RHS		