General Certificate of Education January 2007 Advanced Subsidiary Examination

# MATHEMATICS Unit Pure Core 1

Wednesday 10 January 2007 1.30 pm to 3.00 pm

For this paper you must have:

• an 8-page answer book

• the **blue** AQA booklet of formulae and statistical tables. You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

### Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is not permitted.

### Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

## Advice

• Unless stated otherwise, you may quote formulae, without proof, from the booklet.



MPC1

#### Answer all questions.

**1** The polynomial p(x) is given by

$$p(x) = x^3 - 4x^2 - 7x + k$$

where k is a constant.

- (a) (i) Given that x + 2 is a factor of p(x), show that k = 10. (2 marks)
  - (ii) Express p(x) as the product of three linear factors. (3 marks)
- (b) Use the Remainder Theorem to find the remainder when p(x) is divided by x 3. (2 marks)
- (c) Sketch the curve with equation  $y = x^3 4x^2 7x + 10$ , indicating the values where the curve crosses the *x*-axis and the *y*-axis. (You are **not** required to find the coordinates of the stationary points.) (4 marks)
- 2 The line AB has equation 3x + 5y = 8 and the point A has coordinates (6, -2).
  - (a) (i) Find the gradient of *AB*. (2 marks)
    - (ii) Hence find an equation of the straight line which is perpendicular to *AB* and which passes through *A*. (3 marks)
  - (b) The line *AB* intersects the line with equation 2x + 3y = 3 at the point *B*. Find the coordinates of *B*. (3 marks)
  - (c) The point C has coordinates (2, k) and the distance from A to C is 5. Find the two possible values of the constant k.
    (3 marks)

3 (a) Express 
$$\frac{\sqrt{5}+3}{\sqrt{5}-2}$$
 in the form  $p\sqrt{5}+q$ , where p and q are integers. (4 marks)

- (b) (i) Express  $\sqrt{45}$  in the form  $n\sqrt{5}$ , where *n* is an integer. (1 mark)
  - (ii) Solve the equation

$$x\sqrt{20} = 7\sqrt{5} - \sqrt{45}$$

giving your answer in its simplest form.

(3 marks)

- 4 A circle with centre C has equation  $x^2 + y^2 + 2x 12y + 12 = 0$ .
  - (a) By completing the square, express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$
 (3 marks)

- (b) Write down:
  - (i) the coordinates of C; (1 mark)
  - (ii) the radius of the circle. (1 mark)
- (c) Show that the circle does **not** intersect the *x*-axis. (2 marks)
- (d) The line with equation x + y = 4 intersects the circle at the points P and Q.
  - (i) Show that the x-coordinates of P and Q satisfy the equation

$$x^2 + 3x - 10 = 0 (3 marks)$$

- (ii) Given that P has coordinates (2, 2), find the coordinates of Q. (2 marks)
- (iii) Hence find the coordinates of the midpoint of PQ. (2 marks)

#### Turn over for the next question



The combined internal surface area of the base and four vertical faces is  $54\,\mathrm{m}^2$  .

- (a) (i) Show that  $x^2 + 3xh = 27$ . (2 marks)
  - (ii) Hence express h in terms of x. (1 mark)
  - (iii) Hence show that the volume of water,  $V m^3$ , that the tank can hold when full is given by

$$V = 18x - \frac{2x^3}{3} \tag{1 mark}$$

(b) (i) Find 
$$\frac{dV}{dx}$$
. (2 marks)

(ii) Verify that V has a stationary value when x = 3. (2 marks)

(c) Find  $\frac{d^2 V}{dx^2}$  and hence determine whether V has a maximum value or a minimum value when x = 3. (2 marks)

6 The curve with equation  $y = 3x^5 + 2x + 5$  is sketched below.

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The curve cuts the x-axis at the point A(-1,0) and cuts the y-axis at the point B.

(a) (i) State the coordinates of the point *B* and hence find the area of the triangle *AOB*, where *O* is the origin. (3 marks)

(ii) Find 
$$\int (3x^5 + 2x + 5) \, dx$$
. (3 marks)

(iii) Hence find the area of the shaded region bounded by the curve and the line AB. (4 marks)

- (b) (i) Find the gradient of the curve with equation  $y = 3x^5 + 2x + 5$  at the point A(-1, 0). (3 marks)
  - (ii) Hence find an equation of the tangent to the curve at the point A. (1 mark)
- 7 The quadratic equation  $(k+1)x^2 + 12x + (k-4) = 0$  has real roots.
  - (a) Show that  $k^2 3k 40 \le 0$ . (3 marks)
  - (b) Hence find the possible values of k. (4 marks)

#### END OF QUESTIONS

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