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A-level  
**MATHEMATICS**  
**7357/3**

Paper 3

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Mark scheme

June 2023

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Version: 1.0 Final



Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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## Mark scheme instructions to examiners

### General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

### Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

### Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Working

**AS/A-level Maths/Further Maths assessment objectives**

<b>AO</b>		<b>Description</b>
<b>AO1</b>	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
<b>AO2</b>	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
<b>AO3</b>	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles

### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

### Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

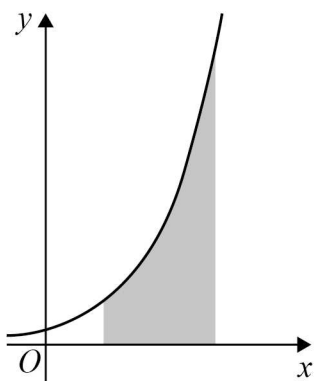
### Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

### Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

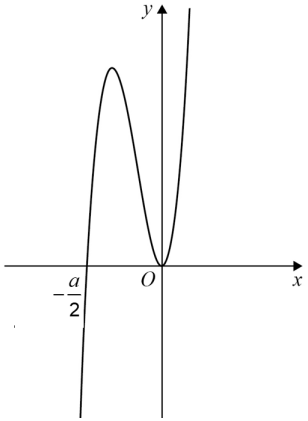
Q	Marking instructions	AO	Marks	Typical solution
1	Ticks correct box	2.2a	R1	$y =  x - 2  - 3$
<b>Question 1 Total</b>			<b>1</b>	

Q	Marking instructions	AO	Marks	Typical solution
2	Ticks correct box	2.2a	R1	
<b>Question 2 Total</b>			<b>1</b>	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles correct answer	2.2a	R1	$x = 3$
<b>Question 3 Total</b>			<b>1</b>	

Q	Marking instructions	AO	Marks	Typical solution
4	Obtains $5x^{-2}$ or $x^{\frac{1}{3}-2}$ <b>PI</b> by $p = -2$ or $q = -\frac{5}{3}$	1.1a	M1	$5x^{-2} - x^{\frac{5}{3}}$
	Obtains $5x^{-2} - x^{\frac{5}{3}}$ <b>PI</b> by $p = -2$ and $q = -\frac{5}{3}$ <b>OE</b>  Allow $-1.67$ or better for $-\frac{5}{3}$  Do not <b>ISW</b> incorrect algebra	1.1b	A1	
<b>Question 4 Total</b>			<b>2</b>	

Q	Marking instructions	AO	Marks	Typical solution
5	Obtains $2 \times 3e^{2x}$ or $6e^{2x}$ or $2y$ <b>PI</b> by correct answer	1.1b	B1	$\frac{dy}{dx} = 2 \times 3e^{2x}$ $y = 10 \Rightarrow \frac{dy}{dx} = 2 \times 10 = 20$
	Substitutes $y = 10$ or $3e^{2x} = 10$ in their $\frac{dy}{dx}$ or substitutes $x = [0.6, 0.602]$ or $x = \frac{1}{2} \ln\left(\frac{10}{3}\right)$ <b>OE</b> in their $\frac{dy}{dx}$	1.1a	M1	
	Obtains 20 <b>CAO</b>  20 cannot come from a rounded value for 20 seen	1.1b	A1	
	<b>Question 5 Total</b>		<b>3</b>	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Draws cubic curve in the correct orientation	1.1a	M1	
	Deduces minimum or maximum at (0,0) on their curve	2.2a	M1	
	Draws a fully correct cubic curve with $x$ -intercept at $-\frac{a}{2}$ shown on the curve	2.2a	A1	
Subtotal			3	

Q	Marking instructions	AO	Marks	Typical solution
6(b)(i)	Substitutes $x = -3$ into $p(x)$  Condone missing bracket for $(-3)^2$  Must see an expression in terms of $a$	1.1a	M1	$(-3)^2(2 \times -3 + a) + 36 = 0$ $-54 + 9a + 36 = 0$ $9a - 18 = 0$ $a = 2$
	Completes reasoned argument with at least one correct intermediate step and no error seen to show $a = 2$ <b>AG</b>  Must set an expression for $p(-3) = 0$  Condone recovery of missing bracket for $(-3)^2$ to get 9  Do not condone any other missing bracket	2.1	R1	
Subtotal			2	



Q	Marking instructions	AO	Marks	Typical solution
6(b)(ii)	States 'translation' or 'translate' or 'translated'	1.1b	B1	Translation $\begin{pmatrix} 0 \\ 36 \end{pmatrix}$
	Must not have other transformation other than translation			
	States the vector $\begin{pmatrix} 0 \\ 36 \end{pmatrix}$ or $36\mathbf{j}$	1.1b	B1	
	<b>Subtotal</b>		<b>2</b>	

Q	Marking instructions	AO	Marks	Typical solution
6(b)(iii)	Explains that the translated graph only has one real solution or only has a root at $-3$	2.4	E1	The translated graph will only have one real solution. $b^2 - 4ac < 0$
	Condone missing 'real'			
	Deduces that the discriminant of $2x^2 + bx + c$ must be negative and shows the required result	2.2a	E1	Hence $b^2 - 4 \times 2 \times c < 0$ $b^2 < 8c$
	Do not allow the use of $a = 2$ with reference to part (b)(i)			
	Allow $b^2 - 8c < 0$ following from $b^2 - 4ac$ seen			
	<b>Subtotal</b>		<b>2</b>	

	<b>Question 6 Total</b>		<b>9</b>	
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Q	Marking instructions	AO	Marks	Typical solution
7(a)	Forms an expression for the area of one sector or both sectors e.g. $\frac{1}{2}r^2\left(\frac{\pi-\theta}{2}\right)$ or $r^2\left(\frac{\pi-\theta}{2}\right)$  or $\frac{1}{2}r^2(\pi-\theta)$  <b>OE</b>  Allow substitution of $r = 2.5$ Condone $r = 5$ Condone missing brackets	3.1b	M1	Area of sectors = $2 \times \frac{1}{2}(2.5)^2\left(\frac{\pi-\theta}{2}\right)$  Area of rhombus = $2 \times \frac{1}{2}(2.5)^2 \sin \theta$  $A = (2.5)^2\left(\frac{\pi-\theta}{2}\right) + (2.5)^2 \sin \theta$  $A = \frac{25}{8}(\pi-\theta) + \frac{25}{4} \sin \theta$
	Forms an expression for the area of half rhombus or full rhombus e.g. $\frac{1}{2}r^2 \sin \theta$ or $r^2 \sin \theta$  Allow substitution of $r = 2.5$ Condone $r = 5$	3.1b	M1	$A = \frac{25}{8}(\pi-\theta + 2 \sin \theta)$
	Substitutes $r = 2.5$ to get a correct expression for area of both sectors or full rhombus  Condone missing brackets	1.1b	A1	
	Completes reasoned argument by calculating correct total area with at least one correct intermediate step and no error seen to show the given result. <b>AG</b>  Allow recovery of missing brackets	2.1	R1	
<b>Subtotal</b>			<b>4</b>	

Q	Marking instructions	AO	Marks	Typical solution
7(b)(i)	Differentiates wrt $\theta$ Condone sign errors and omission of $\frac{25}{8}$	3.1a	M1	$\frac{dA}{d\theta} = -\frac{25}{8} + \frac{25}{4}\cos\theta$ $\text{Max area occurs when } \frac{dA}{d\theta} = 0$ $-\frac{25}{8} + \frac{25}{4}\cos\theta = 0$ $\frac{25}{4}\cos\theta = \frac{25}{8}$ $\cos\theta = \frac{1}{2} \quad \therefore \theta = \frac{\pi}{3}$ $\text{When } \theta = \frac{\pi}{3}$ $\frac{d^2A}{d\theta^2} = -5.41 < 0 \text{ so maximum.}$
	Obtains $\frac{25}{8}(-1 + 2\cos\theta)$ <b>OE</b>	1.1b	A1	
	Explains maximum or stationary or turning occurs when $\frac{dA}{d\theta} = 0$  Label $\frac{dA}{d\theta}$ must be seen	2.4	E1	
	Equates their $\frac{25}{8}(-1 + 2\cos\theta)$ to 0 and rearranges to obtain a value for $\cos\theta$ or $\theta$ when $\cos$ is not seen Condone omission of $\frac{25}{8}$	1.1a	M1	
	Obtains $\cos\theta = \frac{1}{2}$ or $\cos^{-1}\left(\frac{1}{2}\right)$ <b>OE</b> and shows that $\theta = \frac{\pi}{3}$ <b>AG</b>	2.2a	A1	
	Uses second derivative to obtain $-\frac{25\sqrt{3}}{8}$ or <b>AWRT</b> $-5$ and completes argument to show maximum occurs when $\theta = \frac{\pi}{3}$  Allow gradient test  To be awarded R1, marks M1A1M1A1 must be scored as the minimum	2.4	R1	
<b>Subtotal</b>			<b>6</b>	

Q	Marking instructions	AO	Marks	Typical solution
7(b)(ii)	Substitutes $\theta = \frac{\pi}{3}$ into $A = \frac{25}{8}(\pi - \theta + 2\sin \theta)$ fully or <b>AWFW</b> [11.9, 12]	3.4	M1	$A = \frac{25}{8}\left(\pi - \frac{\pi}{3} + 2\sin \frac{\pi}{3}\right)$ $= \frac{25}{8}\left(\frac{2\pi}{3} + \sqrt{3}\right)$
	Obtains the correct exact area ACF with $\sin \frac{\pi}{3}$ evaluated <b>ISW</b>	1.1b	A1	
<b>Subtotal</b>			<b>2</b>	

Q	Marking instructions	AO	Marks	Typical solution
7(c)	States the angle would be the same or the angle will still be $\frac{\pi}{3}$ or (b)(i) stays the same  Condone the answer will be the same	3.5c	E1	The angle would be the same.  The area would be quadrupled.
	States the area would be quadrupled or area is $\frac{25}{2}\left(\frac{2\pi}{3} + \sqrt{3}\right)$ or their answer in (b)(ii) multiplied by 4  <b>OE</b>  Allow (b)(ii) increased by scale factor of 4	3.5c	E1	
<b>Subtotal</b>			<b>2</b>	

<b>Question 7 Total</b>			<b>14</b>	
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Q	Marking instructions	AO	Marks	Typical solution
8	Obtains $5x^4$ <b>PI</b> by $\frac{1}{5}(u-2)^{-\frac{4}{5}}$	1.1b	B1	$u = x^5 + 2$ $\frac{du}{dx} = 5x^4$
	Substitutes for denominator and dx operator <b>PI</b> by fully correct substitution  Condone any limits or missing integral sign or $du$ Condone $dx$ in place $du$	1.1a	M1	$\int \frac{x^9}{u^3} \frac{1}{5x^4} du$ $\frac{1}{5} \int_2^3 \frac{u-2}{u^3} du$ $= \frac{1}{5} \int_2^3 u^{-2} - 2u^{-3} du$
	Substitutes $x^5 = u - 2$ or $x = (u-2)^{\frac{1}{5}}$ in at least one place	1.1a	M1	$\frac{1}{5} [-u^{-1} + u^{-2}]_2^3$
	Obtains $\frac{1}{5} \int \frac{u-2}{u^3} du$  Condone missing or incorrect $\frac{1}{5}$ or any limits  Must have $du$	1.1b	A1	$= \frac{1}{5} \left( \left( \frac{1}{9} - \frac{1}{3} \right) - \left( \frac{1}{4} - \frac{1}{2} \right) \right)$ $= \frac{1}{180}$
	Integrates $u^{-2}$ or $u^{-3}$ correctly	1.1a	M1	
	Obtains $\frac{1}{5} [-u^{-1} + u^{-2}]$ Condone any limits	1.1b	A1	
	Completes reasoned argument by substituting correct limits consistent with their variable to show the given result <b>AG</b>  R1 could be scored if $du$ is missing throughout	2.1	R1	
Question 8 Total			7	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Obtains $y = 10.2$ or $y = \frac{11}{3}$ <b>OE</b> <b>AWFW</b> [3.6, 3.7] for $\frac{11}{3}$	3.4	B1	When $t = 0.2$ , $y = 10.2$ $t = 3$ , $y = \frac{11}{3}$ $10.2 - \frac{11}{3} = 6.53 < 7$ The slide is safe.
	Finds the difference between their two values of $y$	1.1b	M1	
	Makes a comparison between 6.53 and 7 and states that the safety requirement is met.  For 6.53, accept <b>AWFW</b> [6.5, 6.53] <b>OE</b>	3.2a	R1	
	<b>Subtotal</b>		<b>3</b>	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(i)	Obtains $1+t^{-2}$ or $1-2t^{-2}$ <b>OE</b> Ignore labels	1.1b	B1	$\frac{dx}{dt} = 1+t^{-2}$ $\frac{dy}{dt} = 1-2t^{-2}$ $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ $\frac{dy}{dx} = \frac{1-2t^{-2}}{1+t^{-2}}$
	Uses chain rule to obtain $\frac{dy}{dx}$ using their $\frac{dx}{dt}$ and $\frac{dy}{dt}$  Condone missing brackets	1.1a	M1	
	Obtains a correct expression <b>ISW</b>	1.1b	A1	
	<b>Subtotal</b>		<b>3</b>	

Q	Marking instructions	AO	Marks	Typical solution
<b>9(b)(ii)</b>	Forms equation for appropriate derivative equal to zero. Their $\frac{dy}{dx} = 0$ or their $\frac{dy}{dt} = 0$	3.1a	M1	$1 - 2t^{-2} = 0$ $t^2 = 2$ $t = \sqrt{2}$ $y = \sqrt{2} + \frac{2}{\sqrt{2}}$ $= 2\sqrt{2}$ Length of $RS = 2.83$ metres
	Obtains $t = \sqrt{2}$ Allow 1.4 or better for $\sqrt{2}$  $t = \sqrt{2}$ must come from correct $\frac{dy}{dx}$ or $\frac{dy}{dt}$ <b>PI</b> by substituting $\sqrt{2}$ into $y$	1.1b	A1	
	Substitutes their value for $t$ into $y$ and obtains a value for $y$ provided $0.2 < t < 3$	3.4	M1	
	Obtains correct length with unit e.g. $2\sqrt{2}$ metres or 2.8 metres or or <b>AWFW</b> [2.82, 2.83] metres  Allow equivalent correct length in different units  Do not ignore subsequent incorrect rounding	3.2a	A1	
<b>Subtotal</b>			<b>4</b>	

Q	Marking instructions	AO	Marks	Typical solution
<b>9(b)(iii)</b>	States $\tan \theta =$ value of their $\frac{dy}{dx}$ at $t = 3$ <b>OE</b> <b>PI</b> by correct answer or 0.61 or better or $55^\circ$	3.1a	M1	When $t = 3$ , $\frac{dy}{dx} = 0.7$ $\tan \theta = 0.7$ $\theta = 35^\circ$
	Obtains $35^\circ$ <b>CAO</b>	3.2a	A1	
<b>Subtotal</b>			<b>2</b>	

<b>Question 9 Total</b>			<b>12</b>	
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Q	Marking instructions	AO	Marks	Typical solution
10	Circles correct answer	1.1b	B1	$\frac{6}{5}$
<b>Question 10 Total</b>			<b>1</b>	

Q	Marking instructions	AO	Marks	Typical solution
11	Ticks correct box	2.2a	B1	$P(A \cap B) = 0$
<b>Question 11 Total</b>			<b>1</b>	

Q	Marking instructions	AO	Marks	Typical solution
12(a)	<p>States one of the following assumptions in context</p> <ul style="list-style-type: none"> <li>probability of passing the test is constant or 0.4 or fixed or the same or does not change</li> <li>passing the test occurs independently</li> <li>only two outcomes of passing or failing test</li> </ul> <p>Do not ignore incorrect statements about any of the above</p> <p>Must use 'test' Condone 'exam' for 'test'</p> <p>Allow equivalent statements for failing for the reference to probability or independence</p> <p>Do not allow probability being independent</p> <p>Do not allow fixed number of drivers or tests</p>	3.5b	E1	The probability of passing the driving test is constant
<b>Subtotal</b>			<b>1</b>	

Q	Marking instructions	AO	Marks	Typical solution
12(b)	Obtains correct probability <b>AWFW</b> [0.0156, 0.016]	1.1b	B1	0.0157
<b>Subtotal</b>			<b>1</b>	



Q	Marking instructions	AO	Marks	Typical solution
12(c)	Obtains correct probability <b>AWFW</b> [0.908, 0.91]	1.1b	B1	0.908
	<b>Subtotal</b>		<b>1</b>	

Q	Marking instructions	AO	Marks	Typical solution
12(d)	States $P(X \geq 13)$ or $P(13 \leq X \leq 32)$ or $1 - P(X \leq 12)$ or $1 - [0.46, 0.462]$ <b>PI</b> by correct answer	1.1a	M1	$P(X > 12) = 1 - P(X \leq 12)$ $= 1 - 0.4618$ $= 0.538$
	Obtains correct probability <b>AWFW</b> [0.538, 0.54]	1.1b	A1	
	<b>Subtotal</b>		<b>2</b>	

Q	Marking instructions	AO	Marks	Typical solution
12(e)	Obtains 12.8 Do not <b>ISW</b>	1.1b	B1	12.8
	<b>Subtotal</b>		<b>1</b>	

Q	Marking instructions	AO	Marks	Typical solution
12(f)	Uses the correct formula for variance or standard deviation with 32, 0.4 and 0.6 substituted <b>OE</b> <b>PI</b> by 7.68 or <b>AWFW</b> [2.77, 2.8] or $\frac{8\sqrt{3}}{5}$  Ignore incorrect labels Condone missing brackets	1.1a	M1	Variance = $32 \times 0.4 \times 0.6 = 7.68$  Standard deviation = 2.77
	Obtains the correct standard deviation  <b>AWFW</b> [2.77, 2.8] or $\frac{8\sqrt{3}}{5}$ Do not ignore incorrect labels Do not <b>ISW</b> Do not allow $\sqrt{7.68}$	1.1b	A1	
	<b>Subtotal</b>		<b>2</b>	

	<b>Question 12 Total</b>		<b>8</b>	
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Q	Marking instructions	AO	Marks	Typical solution
13(a)	Finds P(both bronze) or P(both silver) or calculates $1 - 2 \times 0.2 \times 0.8$ <b>PI</b> by correct answer	3.1b	M1	$P(\text{both bronze}) = 0.2 \times 0.2 = 0.04$ $P(\text{both silver}) = 0.8 \times 0.8 = 0.64$  $P(\text{both same type}) = 0.68$
	Obtains the correct probability  Ignore incorrect rounding after correct probability seen	1.1b	A1	
	<b>Subtotal</b>		<b>2</b>	

Q	Marking instructions	AO	Marks	Typical solution
13(b)	Finds P(at least one of the coins is bronze)	3.1b	M1	$P(\text{at least one bronze}) = 1 - 0.8 \times 0.8 = 0.36$  $P(\text{both bronze} \mid \text{at least one bronze}) = \frac{0.2 \times 0.2}{0.36} = \frac{1}{9}$
	Obtains the correct probability Allow 0.11 or better for $\frac{1}{9}$ Ignore incorrect rounding after correct probability seen	2.2a	A1	
	<b>Subtotal</b>		<b>2</b>	

	<b>Question 13 Total</b>		<b>4</b>	
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Q	Marking instructions	AO	Marks	Typical solution
14(a)	Obtains 1 or 100%	1.2	B1	1
Subtotal			1	

Q	Marking instructions	AO	Marks	Typical solution
14(b)	States both hypotheses correctly for two-tailed test	2.5	B1	$H_0: \mu = 24\,500$ $H_1: \mu \neq 24\,500$
	Obtains 26 730 or 26 700	1.1a	B1	$\bar{X} = 26\,730$
	States or uses correct model <b>PI</b> by normal with mean 24 500 and variance $\frac{5200^2}{24}$ or 1 126 666.6 or standard deviation $\frac{5200}{\sqrt{24}}$ or 1061 or better <b>OE</b>  or by correct probability <b>AWFW</b> [0.017, 0.02]  or test statistic $(\pm) \frac{\text{their } 26\,730 - 24\,500}{5200 \div \sqrt{24}}$ or or test statistic value <b>AWRT</b> $(\pm)2.1$  or <b>AWRT</b> 22 400 or <b>AWRT</b> 26 600	1.1a	M1	$\bar{X} \sim N(24500, \frac{5200^2}{24})$  $P(\bar{X} > 26\,730) = 0.018$  $0.018 < 0.025$  Reject $H_0$  There is sufficient evidence to suggest that the mean daily mass of aluminium cans recycled has changed.
	Obtains <b>AWFW</b> [0.017, 0.02]  or the correct value of the test statistic <b>AWRT</b> 2.1  or acceptance region <b>AWRT</b> $22\,400 \leq \bar{X} \leq 26\,600$  or critical region $\geq$ <b>AWRT</b> 26 600 ignore reference to the lower region	1.1b	A1	

	allow strict inequalities  or critical value <b>AWRT</b> 26 600 ignore reference to the lower value		
	Correctly compares their probability with 0.025  or correctly compares their positive test statistic with <b>AWRT</b> 1.96  or correctly compares their negative test statistic with <b>AWRT</b> –1.96  or correctly compares 26 730 or 26 700 with their acceptance region or critical region  or correctly compares 26 730 or 26 700 with their upper critical value  May be seen on a diagram	3.5a	M1
	Infers $H_0$ or null hypothesis rejected All figures must be correct  Ignore reference to $H_1$	2.2b	A1
	Concludes correctly in context that there is <b>sufficient evidence</b> to <b>suggest</b> that the <b>mean</b> daily mass of aluminium cans recycled has <b>changed</b> .  To be awarded R1, marks M1A1M1A1 must be scored as the minimum	3.2a	R1
	<b>Subtotal</b>		<b>7</b>

Q	Marking instructions	AO	Marks	Typical solution
14(c)	Explains that a different sample is likely to produce a different sample mean <b>OE</b> e.g sample mean could be the same, sample mean could be different, sample mean will be different  Must refer to the sample mean or mean of the new 24 days	3.5b	E1	Sample mean could be different.  The result could be different so the claim could be wrong.
	Explains that the result in part (b) could be different so the claim is incorrect <b>OE</b> e.g the result might be different so the claim is invalid the result could be the same so the claim is invalid  Statement on the result of the hypothesis test must not be definite	2.2b	E1	
<b>Subtotal</b>			<b>2</b>	

	<b>Question 14 Total</b>		<b>10</b>	
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Q	Marking instructions	AO	Marks	Typical solution
15(a)(i)	Finds IQR <b>PI</b> by correct expression or value for the lower or upper limit	1.1b	B1	$\text{IQR} = 1570 - 1167 = 403$ $1393 - 1.5 \times 403 = 788.5$ $1393 + 1.5 \times 403 = 1997.5$ Hence 2040 should be removed
	Substitutes their IQR and obtains a value for the lower or upper limit <b>PI</b> by correct value for the lower or upper limit	1.1a	M1	
	Obtains correct lower and upper limits and selects mass 2040	3.2a	A1	
<b>Subtotal</b>			<b>3</b>	

Q	Marking instructions	AO	Marks	Typical solution
15(a)(ii)	States 'outlier' <b>ISW</b>	1.2	B1	Outlier
<b>Subtotal</b>			<b>1</b>	

Q	Marking instructions	AO	Marks	Typical solution
15(b)	Forms the equation for total probability <b>PI</b> by $k = 0.08$ <b>OE</b>	3.1b	M1	$0.14 + 0.37 + 0.9k + 0.25 + 0.4k + 1.7k = 1$  $0.76 + 3k = 1$ $k = 0.08$  $P(1 \leq N < 5) = 0.37 + 0.9 \times 0.08 + 0.25 + 0.4 \times 0.08$ $= 0.724$
	Obtains the correct value of $k$ <b>OE</b>	1.1b	A1	
	Forms a correct expression for $P(1 \leq N < 5)$ with or without $k$ substituted e.g $0.37 + 0.9k + 0.25 + 0.4k$ or $0.62 + 1.3k$ or $1 - 0.14 - 1.7k$ <b>OE</b>	1.1a	M1	
	Obtains correct probability	1.1b	A1	
<b>Subtotal</b>			<b>4</b>	

Q	Marking instructions	AO	Marks	Typical solution
15(c)(i)	Identifies the LDS contains cars from 2 years or chooses 100 cars from each year or identifies the LDS contains 5 makes of car or chooses 40 from each make  Condones statement 20 of each car or 10 groups	2.4	M1	Select 20 of each of the five makes of car in each of the two years.
	Concludes that 20 cars selected from each of the 5 makes of car for both years	2.4	R1	
<b>Subtotal</b>			<b>2</b>	

Q	Marking instructions	AO	Marks	Typical solution
15(c)(ii)	States that the disadvantage of quota sampling in LDS is that it is biased or not random or not proportionate.	3.5b	E1	Could produce a biased sample
	<b>Subtotal</b>		<b>1</b>	

	<b>Question 15 Total</b>		<b>11</b>	
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Q	Marking instructions	AO	Marks	Typical solution
16(a)(i)	Obtains correct probability <b>AWFW</b> [0.037, 0.038] Ignore incorrect rounding after correct probability seen	1.1b	B1	0.0375
	<b>Subtotal</b>		<b>1</b>	

Q	Marking instructions	AO	Marks	Typical solution
16(a)(ii)	Obtains correct probability <b>AWFW</b> [0.246, 0.25] Ignore incorrect rounding after correct probability seen	1.1b	B1	0.2467
	<b>Subtotal</b>		<b>1</b>	

Q	Marking instructions	AO	Marks	Typical solution
16(a)(iii)	Obtains correct probability <b>AWFW</b> [0.96, 0.9602] Ignore incorrect rounding after correct probability seen	3.3	B1	0.9601
	<b>Subtotal</b>		<b>1</b>	

Q	Marking instructions	AO	Marks	Typical solution
16(b)	Obtains either $z$ -value from inverse normal distribution <b>AWFW</b> [0.25, 0.26] or <b>AWFW</b> [0.84, 0.85]  Ignore signs	3.1b	B1	$P\left(z < \frac{5.9 - \mu}{\sigma}\right) = 0.6$ $P\left(z > \frac{6.1 - \mu}{\sigma}\right) = 0.2$ $z = 0.2533 \text{ and } z = 0.8416$ $\frac{5.9 - \mu}{\sigma} = 0.2533$ $\frac{6.1 - \mu}{\sigma} = 0.8416$ $\mu = 5.81 \text{ and } \sigma = 0.34$
	Forms an equation with unknown $\mu$ and $\sigma$ using standardised result and $z$ -value for 0.6  Accept $z = \text{AWFW}$ [-4, 4] but do not allow 0, $\pm 0.2$ , $\pm 0.4$ , $\pm 0.6$ or $\pm 0.8$  Condone $\mu - 5.9$ Must use 5.9	3.3	M1	
	Forms an equation with unknown $\mu$ and $\sigma$ using standardised result and $z$ -value for 0.2  Accept $z = \text{AWFW}$ [-4, 4] but do not allow 0, $\pm 0.2$ , $\pm 0.4$ , $\pm 0.6$ or $\pm 0.8$  Condone $\mu - 6.1$ Must use 6.1	3.3	M1	
	Obtains both equations correctly	1.1b	A1	
	Obtains correct value of $\mu$ <b>AWFW</b> [5.8, 5.82] <b>ISW</b>	1.1b	A1	
	Obtains correct value of $\sigma$ <b>AWFW</b> [0.33, 0.35] <b>ISW</b>	1.1b	A1	
	<b>Subtotal</b>		<b>6</b>	
<b>Question 16 Total</b>			<b>9</b>	



Q	Marking instructions	AO	Marks	Typical solution
17	States both hypotheses correctly for one-tailed test 0.7 OE	2.5	B1	$H_0: p = 0.7$ $H_1: p > 0.7$  Under null hypothesis $X \sim B(25, 0.7)$  $P(X \geq 21) = 1 - P(X \leq 20)$ $= 1 - 0.9095$ $= 0.0905$  $0.0905 > 0.025$  Do not reject $H_0$  There is insufficient evidence of an increase in the proportion of local businesses that made a profit in their first year.
	States or uses correct model <b>PI</b> by calculation of one of the probabilities below $P(X \leq 19) = [0.806, 0.807]$ $P(X \leq 20) = [0.909, 0.91]$ $P(X \leq 21) = [0.966, 0.967]$ $P(X \geq 20) = [0.193, 0.1935]$ $P(X \geq 21) = [0.09, 0.091]$ $P(X \geq 22) = [0.033, 0.0333]$ $P(X \geq 23) = [0.0089, 0.00896]$  or critical value of 23 or critical region $\geq 23$  condone missing or incorrect labels	3.3	M1	
	Obtains $[0.09, 0.091]$ or $[0.909, 0.91]$ or obtains critical value 23 or critical region $\geq 23$	1.1b	A1	
	Evaluates binomial model by correctly comparing their $P(X \geq 21)$ or $[0.09, 0.091]$ with 0.025 or evaluates binomial model by correctly comparing their $P(X < 21)$ with 0.975  or evaluates binomial model by correctly determining if 21 is in their critical region	3.5a	M1	
	Infers $H_0$ or null hypothesis not rejected Condone $H_0$ accepted All figures must be correct Ignore reference to $H_1$	2.2b	A1	

	<p>Concludes correctly in context that there is <b>insufficient evidence</b> of an <b>increase</b> in the <b>proportion</b> of local businesses that made a profit in their first year.</p> <p>To be awarded R1, marks M1A1M1A1 must be scored as the minimum</p> <p>Labels of probability calculations must be correct</p> <p>Conclusion must not be definite</p>	3.2a	R1	
	<b>Question 17 Total</b>		<b>6</b>	
	<b>Question Paper Total</b>		<b>100</b>	