

MATHEMATICS
Unit Pure Core 1

MPC1

Wednesday 20 May 2009 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You must **not** use a calculator.



Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

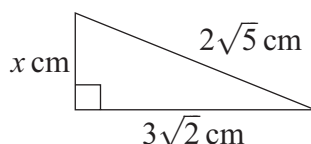
Answer **all** questions.

1 The line AB has equation $3x + 5y = 11$.

- (a) (i) Find the gradient of AB . (2 marks)
- (ii) The point A has coordinates $(2, 1)$. Find an equation of the line which passes through the point A and which is perpendicular to AB . (3 marks)
- (b) The line AB intersects the line with equation $2x + 3y = 8$ at the point C . Find the coordinates of C . (3 marks)

2 (a) Express $\frac{5 + \sqrt{7}}{3 - \sqrt{7}}$ in the form $m + n\sqrt{7}$, where m and n are integers. (4 marks)

(b) The diagram shows a right-angled triangle.



The hypotenuse has length $2\sqrt{5}$ cm. The other two sides have lengths $3\sqrt{2}$ cm and x cm. Find the value of x . (3 marks)

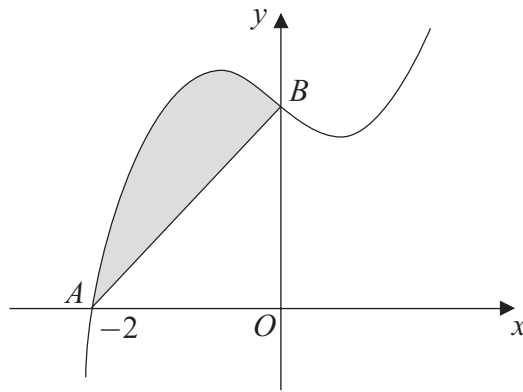
3 The curve with equation $y = x^5 + 20x^2 - 8$ passes through the point P , where $x = -2$.

- (a) Find $\frac{dy}{dx}$. (3 marks)
- (b) Verify that the point P is a stationary point of the curve. (2 marks)
- (c) (i) Find the value of $\frac{d^2y}{dx^2}$ at the point P . (3 marks)
- (ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point. (1 mark)
- (d) Find an equation of the tangent to the curve at the point where $x = 1$. (4 marks)

4 (a) The polynomial $p(x)$ is given by $p(x) = x^3 - x + 6$.

- (i) Find the remainder when $p(x)$ is divided by $x - 3$. (2 marks)
- (ii) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. (2 marks)
- (iii) Express $p(x) = x^3 - x + 6$ in the form $(x + 2)(x^2 + bx + c)$, where b and c are integers. (2 marks)
- (iv) The equation $p(x) = 0$ has one root equal to -2 . Show that the equation has no other real roots. (2 marks)

(b) The curve with equation $y = x^3 - x + 6$ is sketched below.



The curve cuts the x -axis at the point $A(-2, 0)$ and the y -axis at the point B .

- (i) State the y -coordinate of the point B . (1 mark)
- (ii) Find $\int_{-2}^0 (x^3 - x + 6) dx$. (5 marks)
- (iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - x + 6$ and the line AB . (3 marks)

Turn over for the next question

Turn over ►

5 A circle with centre C has equation

$$(x - 5)^2 + (y + 12)^2 = 169$$

(a) Write down:

(i) the coordinates of C ; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(b) (i) Verify that the circle passes through the origin O . *(1 mark)*

(ii) Given that the circle also passes through the points $(10, 0)$ and $(0, p)$, sketch the circle and find the value of p . *(3 marks)*

(c) The point $A(-7, -7)$ lies on the circle.

(i) Find the gradient of AC . *(2 marks)*

(ii) Hence find an equation of the tangent to the circle at the point A , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. *(3 marks)*

6 (a) (i) Express $x^2 - 8x + 17$ in the form $(x - p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Hence write down the minimum value of $x^2 - 8x + 17$. *(1 mark)*

(iii) State the value of x for which the minimum value of $x^2 - 8x + 17$ occurs. *(1 mark)*

(b) The point A has coordinates $(5, 4)$ and the point B has coordinates $(x, 7 - x)$.

(i) Expand $(x - 5)^2$. *(1 mark)*

(ii) Show that $AB^2 = 2(x^2 - 8x + 17)$. *(3 marks)*

(iii) Use your results from part (a) to find the minimum value of the distance AB as x varies. *(2 marks)*

7 The curve C has equation $y = k(x^2 + 3)$, where k is a constant.

The line L has equation $y = 2x + 2$.

- (a) Show that the x -coordinates of any points of intersection of the curve C with the line L satisfy the equation

$$kx^2 - 2x + 3k - 2 = 0 \qquad (1 \text{ mark})$$

- (b) The curve C and the line L intersect in two distinct points.

- (i) Show that

$$3k^2 - 2k - 1 < 0 \qquad (4 \text{ marks})$$

- (ii) Hence find the possible values of k . (4 marks)

END OF QUESTIONS

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