Oxford Cambridge and RSA

Wednesday 8 June 2022 – Afternoon

A Level Further Mathematics A

Y541/01 Pure Core 2

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 **(a)** Find a vector which is perpendicular to both $3\mathbf{i} - 5\mathbf{j} - \mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$. **[1]**

The equations of two lines are $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ and
$\mathbf{r} = \mathbf{i} + 11\mathbf{j} - 4\mathbf{k} + \mu(-\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$.

(b) Show that the lines intersect, stating the point of intersection. **[5]**

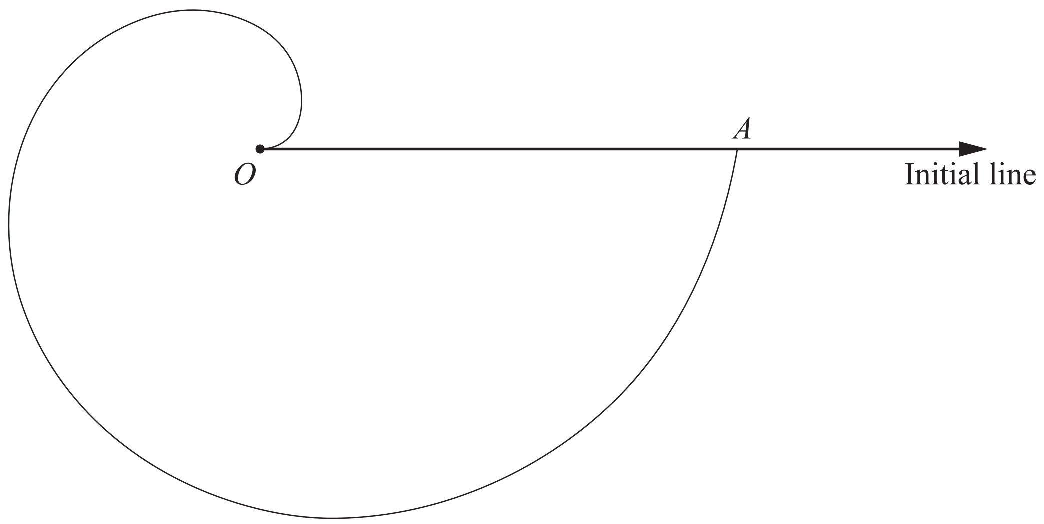
2 Two polar curves, C_1 and C_2, are defined by C_1: $r = 2\theta$ and C_2: $r = \theta + 1$ where $0 \leqslant \theta \leqslant 2\pi$.

C_1 intersects the initial line at two points, the pole and the point A.

(a) Write down the polar coordinates of A. **[2]**

(b) Determine the polar coordinates of the point of intersection of C_1 and C_2. **[2]**

The diagram below shows a sketch of C_1.



(c) On the copy of this sketch in the Printed Answer Booklet, sketch C_2. **[1]**

3 **In this question you must show detailed reasoning.**

The roots of the equation $4x^3 + 6x^2 - 3x + 9 = 0$ are α, β and γ.

Find a cubic equation with integer coefficients whose roots are $\alpha + \beta$, $\beta + \gamma$ and $\gamma + \alpha$. **[6]**

4 **In this question you must show detailed reasoning.**

Determine the smallest value of n for which $\dfrac{1^2+2^2+\ldots+n^2}{1+2+\ldots+n} > 341$. **[4]**

5 **(a)** By using the exponential definitions of $\sinh x$ and $\cosh x$, prove the identity $\cosh 2x \equiv \cosh^2 x + \sinh^2 x$. **[2]**

(b) Hence find an expression for $\cosh 2x$ in terms of $\cosh x$. **[1]**

(c) Determine the solutions of the equation $5\cosh 2x = 16\cosh x + 21$, giving your answers in exact logarithmic form. **[4]**

6 A particle, P, positioned at the origin, O, is projected with a certain velocity along the x-axis. P is then acted on by a single force which varies in such a way that P moves backwards and forwards along the x-axis.

When the time after projection is t seconds, the displacement of P from the origin is x m and its velocity is $v\,\mathrm{m\,s^{-1}}$.

The motion of P is modelled using the differential equation $\ddot{x} + \omega^2 x = 0$, where $\omega\,\mathrm{rad\,s^{-1}}$ is a positive constant.

(a) Write down the general solution of this differential equation. **[1]**

D is the point where $x = d$ for some positive constant, d. When P reaches D it comes to instantaneous rest.

(b) Using the answer to part **(a)**, determine expressions, in terms of ω, d and t only, for the following quantities

- x
- v **[3]**

(c) Hence show that, according to the model, $v^2 = \omega^2(d^2 - x^2)$. **[1]**

The quantity z is defined by $z = \frac{1}{v}$.

(d) Using part **(c)**, determine an expression for z_m, the mean value of z **with respect to the displacement**, as P moves directly from O to D. **[2]**

One measure of the validity of the model is consideration of the value of z_m. If z_m exceeds 8 then the model is considered to be valid.

The value of d is measured as 0.25 to 2 significant figures. The value of ω is measured as 0.75 ± 0.02.

(e) Determine what can be inferred about the validity of the model from the given information. **[1]**

(f) Find, according to the model, the least possible value of the velocity with which P was initially projected. Give your answer to **2** significant figures. **[2]**

7 You are given that a is a parameter which can take only real values.

The matrix **A** is given by $\mathbf{A}=\begin{pmatrix}2 & 4 & -6\\ -3 & 10-4a & 9\\ 7 & 4 & 4\end{pmatrix}$.

(a) Find an expression for the determinant of **A** in terms of a. **[2]**

You are given the following system of equations in x, y and z.

$$\begin{aligned}2x+{}&4y-6z=6\\ -3x+{}&(10-4a)y+9z=-9\\ 7x+{}&4y+4z=11\end{aligned}$$

The system can be written in the form $\mathbf{A}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}6\\ -9\\ 11\end{pmatrix}$.

(b) **(i)** In the case where **A** is **not** singular, solve the given system of equations by using $\mathbf{A}^{-1}$. **[5]**

(ii) In the case where **A is** singular describe the configuration of the planes whose equations are the three equations of the system. **[3]**

The transformation represented by **A** is denoted by T.
A 3-D object of volume $|5a-20|$ is transformed by T to a 3-D image.

(c) **(i)** Determine the range of values of a for which the orientation of the image is the reverse of the orientation of the object. **[1]**

(ii) Determine the range of values of a for which the volume of the image is less than the volume of the object. **[2]**

8 **In this question you must show detailed reasoning.**

It is given that $\sum_{r=k}^{98}\frac{5r+2}{r(r+1)(r+2)}=\frac{20539}{34650}$ for some k.

Determine the value of k. **[7]**

9 **In this question you must show detailed reasoning.**

(a) Show that $\mathrm{Re}\left(\mathrm{e}^{4\mathrm{i}\theta}(\mathrm{e}^{\theta}+\mathrm{e}^{-\mathrm{i}\theta})^4\right)=a\cos 4\theta\cos^4\theta$, where a is an integer to be determined. **[3]**

(b) Hence show that $\cos\frac{1}{12}\pi=\frac{1}{2}\sqrt[4]{b+c\sqrt{3}}$, where b and c are integers to be determined. **[6]**

10 The coordinates of the points A and B are $(3, -2, -1)$ and $(13, 10, 9)$ respectively.

- The plane Π_A contains A and the plane Π_B contains B.
- The planes Π_A and Π_B are parallel.
- The x and y components of any normal to plane Π_A are equal.
- The shortest distance between Π_A and Π_B is 2.

There are **two** possible solution planes for Π_A which satisfy the above conditions.

Determine the acute angle between these two possible solution planes. **[8]**

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