OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4723

Core Mathematics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Monday 1 June 2009
Morning

Duration: 1 hour 30 minutes

4723

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1

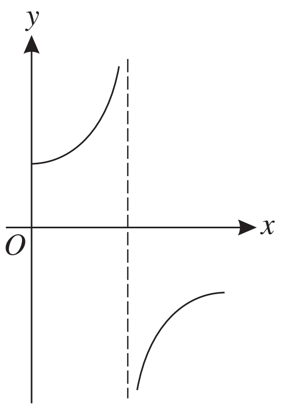


Fig. 1

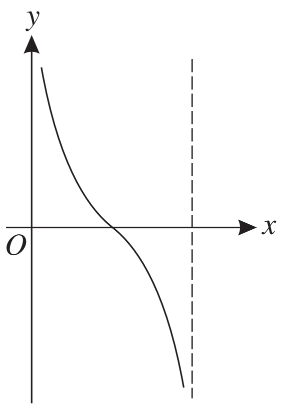


Fig. 2

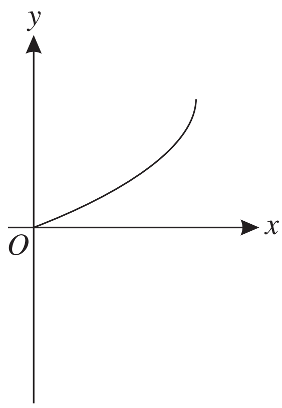


Fig. 3

Each diagram above shows part of a curve, the equation of which is one of the following:

$$y = \sin^{-1} x, \quad y = \cos^{-1} x, \quad y = \tan^{-1} x, \quad y = \sec x, \quad y = \operatorname{cosec} x, \quad y = \cot x.$$

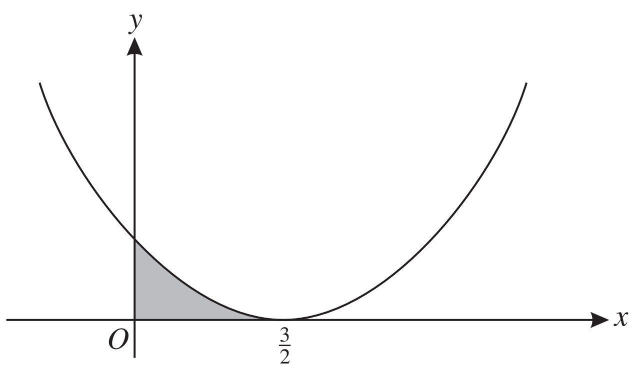
State which equation corresponds to

(i) Fig. 1, **[1]**

(ii) Fig. 2, **[1]**

(iii) Fig. 3. **[1]**

2



The diagram shows the curve with equation $y = (2x - 3)^2$. The shaded region is bounded by the curve and the lines $x = 0$ and $y = 0$. Find the exact volume obtained when the shaded region is rotated completely about the x-axis. **[5]**

3 The angles α and β are such that

$$\tan \alpha = m + 2 \quad \text{and} \quad \tan \beta = m,$$

where m is a constant.

(i) Given that $\sec^2 \alpha - \sec^2 \beta = 16$, find the value of m. **[3]**

(ii) Hence find the exact value of $\tan(\alpha + \beta)$. **[3]**

4 It is given that $\int_a^{3a} (e^{3x} + e^x)\,dx = 100$, where a is a positive constant.

(i) Show that $a = \frac{1}{9}\ln(300 + 3e^a - 2e^{3a})$. **[5]**

(ii) Use an iterative process, based on the equation in part **(i)**, to find the value of a correct to 4 decimal places. Use a starting value of 0.6 and show the result of each step of the process. **[4]**

5 The functions f and g are defined for all real values of x by

$$f(x) = 3x - 2 \quad \text{and} \quad g(x) = 3x + 7.$$

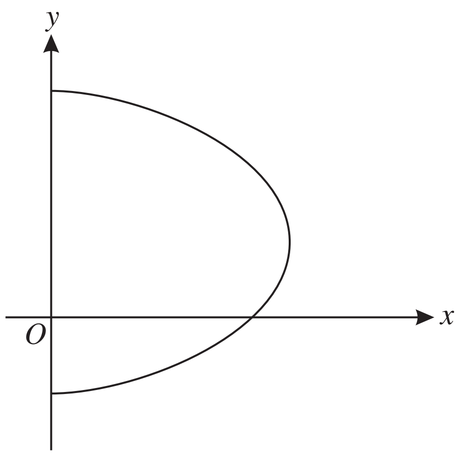
Find the exact coordinates of the point at which

(i) the graph of $y = fg(x)$ meets the x-axis, **[3]**

(ii) the graph of $y = g(x)$ meets the graph of $y = g^{-1}(x)$, **[3]**

(iii) the graph of $y = |f(x)|$ meets the graph of $y = |g(x)|$. **[4]**

6



The diagram shows the curve with equation $x = (37 + 10y - 2y^2)^{\frac{1}{2}}$.

(i) Find an expression for $\frac{dx}{dy}$ in terms of y. **[2]**

(ii) Hence find the equation of the tangent to the curve at the point (7, 3), giving your answer in the form $y = mx + c$. **[5]**

7 **(i)** Express $8\sin\theta - 6\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[3]**

(ii) Hence

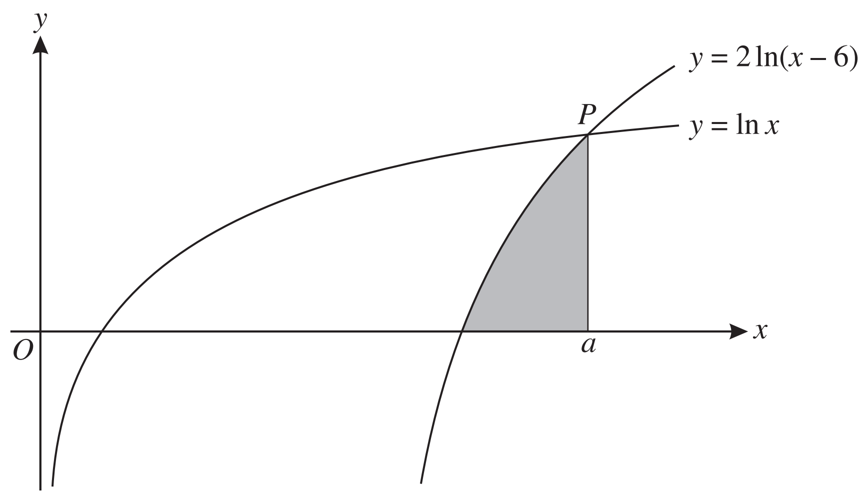
(a) solve, for $0° < \theta < 360°$, the equation $8\sin\theta - 6\cos\theta = 9$, **[4]**

(b) find the greatest possible value of

$$32\sin x - 24\cos x - (16\sin y - 12\cos y)$$

as the angles x and y vary. **[3]**

8



The diagram shows the curves $y = \ln x$ and $y = 2\ln(x-6)$. The curves meet at the point P which has x-coordinate a. The shaded region is bounded by the curve $y = 2\ln(x-6)$ and the lines $x = a$ and $y = 0$.

(i) Give details of the pair of transformations which transforms the curve $y = \ln x$ to the curve $y = 2\ln(x-6)$. **[3]**

(ii) Solve an equation to find the value of a. **[4]**

(iii) Use Simpson's rule with two strips to find an approximation to the area of the shaded region. **[3]**

9 **(a)** Show that, for all non-zero values of the constant k, the curve

$$y = \frac{kx^2 - 1}{kx^2 + 1}$$

has exactly one stationary point. **[5]**

(b) Show that, for all non-zero values of the constant m, the curve

$$y = e^{mx}(x^2 + mx)$$

has exactly two stationary points. **[7]**