OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced Subsidiary GCE **4761**

Mechanics 1

Mark Scheme for June 2010

Q 1		mark	notes
(i)	$v^2 = 0^2 + 2\times 9.8\times 0.75$	M1	Use of $v^2 = u^2 + 2as$ with $u = 0$ and $a = \pm g$. Accept muddled units and sign errors.
		A1	Allow wrong or wrongly converted units not sign errors
	$v = \pm 3.8340...$ so 3.83 m s^{-1} (3. s. f.)	A1	cao [SC2 for 38.3… seen WWW and SC3 for 3.83… seen WWW]
		3	
		3	

Q 2		mark	notes
(i)	Resolving	M1	Resolving in at least 1 of horiz or vert. Accept $\sin \leftrightarrow \cos$. No extra terms.
	$\leftarrow\ 250\sin 70 = 234.92...$ so 235 N (3 s. f.)	A1	Either both expressions correct (neglect direction) or one correct in correct direction
	$\uparrow\ 250\cos 70 = 85.5050...$ so 85.5 N (3 s. f.)	A1	cao Both evaluated and directions correct
		3	
(ii)	$250 \div 2 = 125$ N	B1	Accept $125g$ only if tension taken to be $250g$ in (i)
		1	
		4	

Q 3		mark	notes
(i)	$\begin{pmatrix}-1\\14\\-8\end{pmatrix} + \begin{pmatrix}3\\-9\\10\end{pmatrix} + \mathbf{F} = 4\begin{pmatrix}-1\\2\\4\end{pmatrix}$	M1	N2L. Allow sign errors in applying N2L. Do not condone $\mathbf{F} = m\mathbf{g}$**a.** Allow one given force omitted.
		M1	Attempt to add $\begin{pmatrix}-1\\14\\-8\end{pmatrix}$ and $\begin{pmatrix}3\\-9\\10\end{pmatrix}$
	$\mathbf{F} = \begin{pmatrix}-6\\3\\14\end{pmatrix}$	A1 A1	Two components correct cao
		4	
(ii)	$\mathbf{v} = \begin{pmatrix}-3\\3\\6\end{pmatrix} + 3\begin{pmatrix}-1\\2\\4\end{pmatrix} = \begin{pmatrix}-6\\9\\18\end{pmatrix}$ so $\begin{pmatrix}-6\\9\\18\end{pmatrix}$ m s^{-1}.	M1	$\mathbf{v} = \mathbf{u} + t\mathbf{a}$ with given **u** and **a**. Could go via **s**. If integration used, require arbitrary constant (need not be evaluated)
		A1	cao isw
	speed is $\sqrt{(-6)^2 + 9^2 + 18^2} = 21$ m s^{-1}.	M1	Allow -6^2 even if interpreted as – 36. Only FT **their v**.
		F1	FT their **v** only. [Award M1 F1 for 21 seen WWW]
		4	
		8	

Q 4		mark	notes
(i)	Diagram for P or Q Other diagram	B1 B1 2	Must be properly labelled with arrows Must be properly labelled with arrows consistent with 1st diagram Accept single diagram if clear.
(ii)	Let tension in rope be T N and accn ↑ a m s^{-2} For box P: N2L ↑ $1030 - 75g - T = 75a$ For box Q: N2L ↑ $T - 25g = 25a$	M1 A1 A1 3	N2L applied correctly to either part. Allow $F = mga$ and sign errors. Do not condone missing or extra forces. Direction of a consistent with equation for P. [Condone taking + ve downwards in either equation. +ve direction must be consistent in both equations to receive both A1s]
(iii)	tension is 257.5 N	M1 A1 2	Solving for T **their** simultaneous equations with 2 variables. cao CWO
		7	

Q 5		mark	notes
(i)	$270 - \arctan\left(\frac{6}{4}\right)$ = 213.69… so 214°	M1 A1 2	Award for arctan p seen where $p = \pm\frac{6}{4}$ or $\frac{4}{6}$, or equivalent cao
(ii)	Need $(-4 + 3k)\mathbf{i} + (-6 - 2k)\mathbf{j} = \lambda(7\mathbf{i} - 9\mathbf{j})$ * **either** so $\frac{-4+3k}{-6-2k} = \frac{7}{-9}$. or equivalent $k = 6$ **or** $-4 + 3k = 7\lambda$ $-6 - 2k = -9\lambda$ $k = 6$ **trial and error method**	M1 M1 A1 A1 M1 A1 A1 4	Attempt to get LHS in the direction of $(7\mathbf{i} - 9\mathbf{j})$. Could be done by finding (tangents of) angles. Accept the use of $\lambda = 1$. Attempt to solve **their** *. Allow = $\frac{7}{9}, \frac{9}{7}, -\frac{9}{7}$ Expression correct Award full marks for $k = 6$ found WWW Attempt to solve **their** *. Must have both equations. Correct equations Award full marks for $k = 6$ found WWW M1 any attempt to find the value of k and 'test' M1 Systematic attempt in (the equivalent of) **their** * Award full marks for $k = 6$ found WWW
		6	

Q6		mark	notes
(i)	Vertically		
.	$y = 8t - 4.9t^2$	M1	Use of $s = ut + 0.5at^2$ with $g = \pm 9.8,\ \pm 10$. Accept u = 0 or 14.4… or 14.4 $\sin\theta$ or $u\sin\theta$ but not 12 . Allow use of + 3.6.
		A1	Accept derivation of – 4.9 not clear. cao.
	Horizontally $x = 12t$	B1	
		3	
(ii)	**either**		
	Require $y = -3.6$	M1	Equating **their** y to ±3.6 or equiv. Any form.
	so $-3.6 = 8t - 4.9t^2$		
	Use of formula or $4.9(t-2)(t+\frac{18}{49}) = 0$	M1	A method for solving a 3 term quadratic to give at least 1 root. Allow **their** y and re-arrangement errors.
	Roots are 2 and $-\frac{18}{49}$ (= – 0.367346…)	A1	WWW. Accept no reference to 2nd root [Award SC3 for t = 2 seen WWW]
	Horizontal distance is 12 × 2 = 24	M1	FT **their** x and t.
	so 24 m	F1	FT only **their** t (as long as it is +ve and is not obtained with sign error(s) e.g. –ve sign just dropped)
	or		
	Require $y = -3.6$	M1	Equating **their** y to ±3.6 or equiv. Any form.
	so $-3.6 = 8t - 4.9t^2$		
	Eliminate t between		
	$x = 12t$ and $-3.6 = 8t - 4.9t^2$	M1	Expressions in any form. Elimination must be complete
	so $0 = 3.6 + \frac{8x}{12} - \frac{4.9x^2}{144}$	A1	Accept in any form. May be implied.
	Use of formula or factorise	M1	A method for solving a 3 term quadratic to give at least 1 root. Allow **their** y and re-arrangement errors.
	+ve root is 24 so 24m	F1	FT from **their** quadratic after re-arrangement. Must be +ve.
	or		
	Methods that divide the motion into sections		
	Projection to highest point (A)		
	Highest point to level of jetty (B)		
	Level of jetty to sea (C)		
	Combination of A, B and C may be used	M1	Attempt to find times or distances for sections that give the total horizontal distance travelled
		M1	Correct method for one section to find time or distance
	(A) 0.8163.. s; 9.7959.. m: (B) 0.816…s; 9.7959.. m (C): 0.3673… s; 4.4081… m	A1	Any time or distance for a section correct
		A1	2nd time or distance correct (The two sections must not be A and B)
		A1	cao
		5	
		8	

Q 7		mark	notes
(i)			
(A)	4 m	B1	
(B)	$12 - (-4) = 16$ m	M1	Looking for distance. Need evidence of taking account of +ve and –ve displacements.
		A1	
(C)	$1 < t < 3.5$	B1	The values 1 and 3.5
		B1	Strict inequality
(D)	$t = 1$, $t = 3.5$	B1	Do not award if extra values given.
		6	
(ii)	$v = -8t + 8$	M1	Differentiating
		A1	
	$a = -8$	F1	
		3	
(iii)	$-8t + 8 = 4$ so $t = 0.5$ so 0.5 s	B1	FT **their** v.
	$-8t + 8 = -4$ so $t = 1.5$ so 1.5 s	B1	FT **their** v.
		2	
(iv)	**method 1**		
	Need velocity at $t = 3$		
	$v(3) = -8 \times 3 + 8 = -16$	B1	FT **their** v from (ii)
	either		
	$v = \int 32 \, dt = 32t + C$	M1	Accept $32t + C$ or $32t$. SC1 if $\int_3^4 32 dt$ attempted.
	$v = -16$ when $t = 3$ gives $v = 32t - 112$	A1	Use of **their** -16 from an attempt at v when t=3
	$y = \int (32t - 112) dt = 16t^2 - 112t + D$	M1	FT **their** v of the form $pt + q$ with $p \neq 0$ and $q \neq 0$.
			Accept if at least 1 term correct. Accept no D.
	$y = 0$ when $t = 3$		
	gives $y = 16t^2 - 112t + 192$	A1	cao.
	or		
	$y = -16 \times (t-3) + \frac{1}{2} \times 32 \times (t-3)^2$	M1	Use of $s = ut + \frac{1}{2}at^2$
		A1	Use of **their** -16 (not 0) from an attempt at v when t=3 and 32. Condone use of just t
		M1	Use of $t \pm 3$
		A1	cao
	(so $y = 16t^2 - 112t + 192$)		
	method 2		
	Since accn is constant, the displacement y is a quadratic function. Since we have $y = 0$ at $t = 3$ and $t = 4$		
	$y = k(t-3)(t-4)$	M1	Use of a quadratic function (condone no k)
		A1	Correct use of roots
		B1	k present
	When $t = 3.5$, $y = -4$		
	so $-4 = k \times \frac{1}{2} \times -\frac{1}{2}$	M1	Or consider velocity at $t = 3$
	so $k = 16$ (and $y = 16t^2 - 112t + 192$)	A1	cao. Accept k without y simplified.
		5	
		16	

Q8		mark	notes
(i)	N2L **i** direction $150 = 250a$ $a = 0.6$ so 0.6 m s^{-2}	 M1 A1 2	 Use of N2L. Allow $F = mga$. Accept no reference to direction
(ii)	150 N – **i** direction	B1 B1 2	 Allow correct description or arrow [Accept '– 150 in **i** direction' for B1 B1]
(iii)	For force only in direction perp to **i** $300\sin 40 = 450\sin\theta$ $\theta = 25.37300\ldots$ so 25.4° (3 s. f.) In **i** direction $300\cos 40 + 150 + 450\cos\theta$ 786.4017… so 786**i** N (3 s. f.)	 M1 B1 A1 M1 A1 6	 Resolution of both terms attempted. Allow sin ↔ cos if in both terms. Allow 250 or 250g present. 300sin40 or 450sinθ Accept ± . Accept answer rounding to 25.5. Allow SC1 if seen in this part. Proper resolution attempted of 450 **and** 300. Allow sin ↔ cos if in both terms Accept use of **their** θ or just θ. Either resolution correct. Accept **their** θ or just θ. Accept sin/cos consistent with use for cpt perpendicular to **i.** Accept no reference to direction cao. Allow SC1 WW
(iv)	Using $s = ut + 0.5at^2$ $1 = 0.5a \times 2^2$ $a = 0.5$ Using N2L in **i** direction 786.4017… $– F = 250 \times 0.5$ 661.4017… so 661 N (3 s. f.)	 M1 A1 M1 A1 E1 5	 Appropriate (sequence of) *suvat* [WW M0 A0] Use of $F = ma$ with **their** 786.4 and **their** a. No extra forces. Allow sign errors. All correct using **their** 786.4 and a Use of N2L clearly shown. (Accept 0.5 used WW)
(v)	Using N2L in **i** direction **either** $125 - 200 = 250a_1$ **or** (starting again) $786.4017\ldots – (200 + 661.4017\ldots) = 250\ a_1$ so $a_1 = – 0.3$ Using $v^2 = u^2 + 2\ a_1 s$ $v^2 = 1.8^2 + 2 \times (– 0.3) \times 1.65$ $v = 1.5$ so 1.5 m s^{-1}	 M1 F1 M1 F1 A1 5	 Use of $F = ma$ with **their** values. Allow 1 force missing FT only **their** 786… and **their** 661 Appropriate (sequence of) *suvat* with $u \neq 0$. Must be 'new' a obtained by using N2L. Only FT use of ± **their** a_1 cao
		20	