

A Level Further Mathematics A

Y541 Pure Core 2

Sample Question Paper

Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 30 minutes

You must have:

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:

- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 Find $\sum_{r=1}^n (r+1)(r+5)$. Give your answer in a fully factorised form. [4]

2 **In this question you must show detailed reasoning.**

The finite region R is enclosed by the curve with equation $y = \frac{8}{\sqrt{16+x^2}}$, the x-axis and the lines $x=0$ and $x=4$. Region R is rotated through 360° about the x-axis. Find the exact value of the volume generated. [4]

3 (i) Find $\sum_{r=1}^n \left(\frac{1}{r} - \frac{1}{r+2} \right)$. [3]

(ii) What does the sum in part (i) tend to as $n \rightarrow \infty$? Justify your answer. [1]

4 It is given that $\frac{5x^2+x+12}{x^3+kx} \equiv \frac{A}{x} + \frac{Bx+C}{x^2+k}$ where k, A, B and C are positive integers. Determine the set of possible values of k. [5]

5 **In this question you must show detailed reasoning.**

Evaluate $\int_0^\infty 2xe^{-x} dx$.

[You may use the result $\lim_{x \rightarrow \infty} xe^{-x} = 0$.] [4]

6 The equation of a plane Π is $x - 2y - z = 30$.

(i) Find the acute angle between the line $\mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 3 \\ 2 \end{pmatrix}$ and Π . [4]

(ii) Determine the geometrical relationship between the line $\mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$ and Π . [4]

7 (i) Use the Maclaurin series for $\sin x$ to work out the series expansion of $\sin x \sin 2x \sin 4x$ up to and including the term in x^5 . [4]

(ii) Hence find, in exact surd form, an approximation to the least positive root of the equation $2 \sin x \sin 2x \sin 4x = x$. [3]

- 8 The equation of a curve is $y = \cosh^2 x - 3\sinh x$. Show that $\left(\ln\left(\frac{3+\sqrt{13}}{2}\right), -\frac{5}{4}\right)$ is the only stationary point on the curve. [8]

- 9 A curve has equation $x^4 + y^4 = x^2 + y^2$, where x and y are not both zero.

(i) Show that the equation of the curve in polar coordinates is $r^2 = \frac{2}{2 - \sin^2 2\theta}$. [4]

- (ii) Deduce that no point on the curve $x^4 + y^4 = x^2 + y^2$ is further than $\sqrt{2}$ from the origin. [2]

- 10 Let $C = \sum_{r=0}^{20} \binom{20}{r} \cos r\theta$. Show that $C = 2^{20} \cos^{20}\left(\frac{1}{2}\theta\right) \cos 10\theta$. [8]

- 11 During an industrial process substance X is converted into substance Z. Some of the substance X goes through an intermediate phase, and is converted to substance Y, before being converted to substance Z. The situation is modelled by

$$\frac{dy}{dt} = 0.3x - 0.2y \quad \text{and} \quad \frac{dz}{dt} = 0.2y + 0.1x$$

where x , y and z are the amounts in kg of X, Y and Z at time t hours after the process starts.

Initially there is 10 kg of substance X and nothing of substances Y and Z. The amount of substance X decreases exponentially. The initial rate of decrease is 4 kg per hour.

- (i) Show that $x = Ae^{-0.4t}$, stating the value of A. [3]

(ii) (a) Show that $\frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt} = 0$. [2]

- (b) Comment on this result in the context of the industrial process. [2]

- (iii) Express y in terms of t . [5]

- (iv) Determine the maximum amount of substance Y present during the process. [3]

- (v) How long does it take to produce 9 kg of substance Z? [2]

END OF QUESTION PAPER

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