General Certificate of Education
Advanced Subsidiary Examination
January 2010

Mathematics MPC2

Unit Pure Core 2

Monday 11 January 2010 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

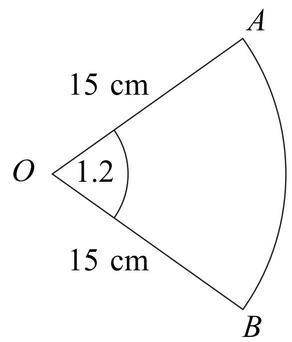
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The diagram shows a sector OAB of a circle with centre O.

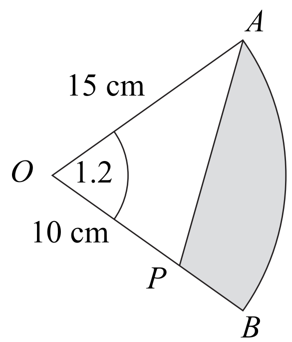


The radius of the circle is 15 cm and angle $AOB = 1.2$ radians.

(a) (i) Show that the area of the sector is $135\ \text{cm}^2$. *(2 marks)*

(ii) Calculate the length of the arc AB. *(2 marks)*

(b) The point P lies on the radius OB such that $OP = 10$ cm, as shown in the diagram below.



Calculate the perimeter of the shaded region bounded by AP, PB and the arc AB, giving your answer to three significant figures. *(5 marks)*

2 At the point (x, y) on a curve, where $x > 0$, the gradient is given by

$$\frac{dy}{dx} = 7\sqrt{x^5} - 4$$

(a) Write $\sqrt{x^5}$ in the form x^k, where k is a fraction. *(1 mark)*

(b) Find $\int \left(7\sqrt{x^5} - 4\right) dx$. *(3 marks)*

(c) Hence find the equation of the curve, given that the curve passes through the point $(1, 3)$. *(3 marks)*

3 (a) Find the value of x in each of the following:

(i) $\log_9 x = 0$; *(1 mark)*

(ii) $\log_9 x = \frac{1}{2}$. *(1 mark)*

(b) Given that

$$2 \log_a n = \log_a 18 + \log_a (n - 4)$$

find the possible values of n. *(5 marks)*

4 An arithmetic series has first term a and common difference d.

The sum of the first 31 terms of the series is 310.

(a) Show that $a + 15d = 10$. *(3 marks)*

(b) Given also that the 21st term is twice the 16th term, find the value of d. *(3 marks)*

(c) The nth term of the series is u_n. Given that $\sum_{n=1}^{k} u_n = 0$, find the value of k. *(4 marks)*

Turn over ▶

5 A curve has equation $y = \frac{1}{x^3} + 48x$.

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Hence find the equation of each of the two tangents to the curve that are parallel to the x-axis. *(4 marks)*

(c) Find an equation of the normal to the curve at the point $(1, 49)$. *(3 marks)*

6 (a) Sketch the curve with equation $y = 2^x$, indicating the coordinates of any point where the curve intersects the coordinate axes. *(2 marks)*

(b) (i) Use the trapezium rule with five ordinates (four strips) to find an approximate value for $\int_0^2 2^x \, dx$, giving your answer to three significant figures. *(4 marks)*

(ii) State how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

(c) Describe a geometrical transformation that maps the graph of $y = 2^x$ onto the graph of $y = 2^{x+7} + 3$. *(3 marks)*

(d) The curve $y = 2^{x+k} + 3$ intersects the y-axis at the point $A(0, 8)$.

Show that $k = \log_m n$, where m and n are integers. *(2 marks)*

7 (a) The first four terms of the binomial expansion of $(1 + 2x)^7$ in ascending powers of x are $1 + ax + bx^2 + cx^3$. Find the values of the integers a, b and c. *(4 marks)*

(b) Hence find the coefficient of x^3 in the expansion of $\left(1 - \frac{1}{2}x\right)^2 (1 + 2x)^7$. *(4 marks)*

8 (a) Solve the equation $\tan(x + 52°) = \tan 22°$, giving the values of x in the interval $0° \leqslant x \leqslant 360°$. *(3 marks)*

(b) (i) Show that the equation

$$3\tan\theta = \frac{8}{\sin\theta}$$

can be written as

$$3\cos^2\theta + 8\cos\theta - 3 = 0$$ *(3 marks)*

(ii) Find the value of $\cos\theta$ that satisfies the equation

$$3\cos^2\theta + 8\cos\theta - 3 = 0$$ *(2 marks)*

(iii) Hence solve the equation

$$3\tan 2x = \frac{8}{\sin 2x}$$

giving all values of x to the nearest degree in the interval $0° \leqslant x \leqslant 180°$. *(4 marks)*

END OF QUESTIONS

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