## ADVANCED GCE MATHEMATICS (MEI)

## Mechanics 2

## Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

- Scientific or graphical calculator


## Tuesday 15 June 2010 Morning

Duration: 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer all the questions.
- Do not write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \mathrm{~m} \mathrm{~s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g=9.8$.


## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72 .
- This document consists of 8 pages. Any blank pages are indicated.

1 Two sledges P and Q , with their loads, have masses of 200 kg and 250 kg respectively and are sliding on a horizontal surface against negligible resistance. There is an inextensible light rope connecting the sledges that is horizontal and parallel to the direction of motion.

Fig. 1 shows the initial situation with both sledges travelling with a velocity of $5 \mathbf{i} \mathrm{~m} \mathrm{~s}^{-1}$.


Fig. 1

A mechanism on Q jerks the rope so that there is an impulse of $250 \mathbf{i N s}$ on P .
(i) Show that the new velocity of P is $6.25 \mathrm{im} \mathrm{s}^{-1}$ and find the new velocity of Q .

There is now a direct collision between the sledges and after the impact P has velocity $4.5 \mathbf{i} \mathrm{~m} \mathrm{~s}^{-1}$.
(ii) Show that the velocity of Q becomes $5.4 \mathrm{i} \mathrm{m} \mathrm{s}^{-1}$.

Calculate the coefficient of restitution in the collision.

Before the rope becomes taut again, the velocity of $P$ is increased so that it catches up with $Q$. This is done by throwing part of the load from sledge P in the $-\mathbf{i}$ direction so that P 's velocity increases to $5.5 \mathbf{i ~ m ~ s}{ }^{-1}$. The part of the load thrown out is an object of mass 20 kg .
(iii) Calculate the speed of separation of the object from $P$.

When the sledges directly collide again they are held together and move as a single object.
(iv) Calculate the common velocity of the pair of sledges, giving your answer correct to 3 significant figures.


Fig. 2
Fig. 2 shows a stand on a horizontal floor and horizontal and vertical coordinate axes $\mathrm{O} x$ and $\mathrm{O} y$. The stand is modelled as

- a thin uniform rectangular base PQRS, 30 cm by 40 cm with mass 15 kg ; the sides QR and PS are parallel to $\mathrm{O} x$,
- a thin uniform vertical rod of length 200 cm and mass 3 kg that is fixed to the base at O , the mid-point of PQ and the origin of coordinates,
- a thin uniform top rod AB of length 50 cm and mass 2 kg ; AB is parallel to $\mathrm{O} x$.

Coordinates are referred to the axes shown in the figure.
(i) Calculate the coordinates of the centre of mass of the stand.

A small object of mass 5 kg is fixed to the rod AB at a distance of 40 cm from A .
(ii) Show that the coordinates of the centre of mass of the stand with the object are $(22,68)$.

The stand is tilted about the edge PQ until it is on the point of toppling. The angle through which the stand is tilted is called 'the angle of tilt'. This procedure is repeated about the edges QR and RS.
(iii) Making your method clear, determine which edge requires the smallest angle of tilt for the stand to topple.

The small object is removed. A light string is attached to the stand at A and pulled at an angle of $50^{\circ}$ to the downward vertical in the plane Oxy in an attempt to tip the stand about the edge RS.
(iv) Assuming that the stand does not slide, find the tension in the string when the stand is about to turn about the edge RS.

3 Fig. 3 shows a framework in a vertical plane constructed of light, rigid rods AB, BC, CD, DA and $B D$. The rods are freely pin-jointed to each other at $A, B, C$ and $D$ and to a vertical wall at $A$. $A B C D$ is a parallelogram with AD horizontal and BD vertical; the dimensions of the framework, in metres, are shown. There is a vertical load of 300 N acting at C and a vertical wire attached to D , with tension $T \mathrm{~N}$, holds the framework in equilibrium. The horizontal and vertical forces, $X \mathrm{~N}$ and $Y \mathrm{~N}$, acting on the framework at A due to the wall are also shown.


Fig. 3
(i) Show that $T=600$ and calculate the values of $X$ and $Y$.
(ii) Draw a diagram showing all the forces acting on the framework, and also the internal forces in the rods.
(iii) Calculate the internal forces in the five rods, indicating whether each rod is in tension or compression (thrust). (You may leave answers in surd form. Your working in this part should correspond to your diagram in part (ii).)

Suppose that the vertical wire is attached at B instead of D and that the framework is still in equilibrium.
(iv) Without doing any further calculations, state which four of the rods have the same internal forces as in part (iii) and say briefly why this is the case. Determine the new force in the fifth rod.

4 A box of mass 16 kg is on a uniformly rough horizontal floor with an applied force of fixed direction but varying magnitude $P \mathrm{~N}$ acting as shown in Fig. 4. You may assume that the box does not tip for any value of $P$. The coefficient of friction between the box and the floor is $\mu$.


Fig. 4

Initially the box is at rest and on the point of slipping with $P=58$.
(i) Show that $\mu$ is about 0.25 .

In the rest of this question take $\mu$ to be exactly 0.25 .
The applied force on the box is suddenly increased so that $P=70$ and the box moves against friction with the floor and another horizontal retarding force, $S$. The box reaches a speed of $1.5 \mathrm{~m} \mathrm{~s}^{-1}$ from rest after 5 seconds; during this time the box slides 3 m .
(ii) Calculate the work done by the applied force of 70 N and also the average power developed by this force over the 5 seconds.
(iii) By considering the values of time, distance and speed, show that an object starting from rest that travels 3 m while reaching a speed of $1.5 \mathrm{~m} \mathrm{~s}^{-1}$ in 5 seconds cannot be moving with constant acceleration.

The reason that the acceleration is not constant is that the retarding force $S$ is not constant.
(iv) Calculate the total work done by the retarding force $S$.

