

ADVANCED GCE

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

4753/01

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Thursday 15 January 2009
Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

- 1 Solve the inequality $|x - 1| < 3$. [3]
- 2 (i) Differentiate $x \cos 2x$ with respect to x . [3]
 (ii) Integrate $x \cos 2x$ with respect to x . [4]
- 3 Given that $f(x) = \frac{1}{2} \ln(x - 1)$ and $g(x) = 1 + e^{2x}$, show that $g(x)$ is the inverse of $f(x)$. [3]
- 4 Find the exact value of $\int_0^2 \sqrt{1 + 4x} \, dx$, showing your working. [5]
- 5 (i) State the period of the function $f(x) = 1 + \cos 2x$, where x is in degrees. [1]
 (ii) State a sequence of two geometrical transformations which maps the curve $y = \cos x$ onto the curve $y = f(x)$. [4]
 (iii) Sketch the graph of $y = f(x)$ for $-180^\circ < x < 180^\circ$. [3]
- 6 (i) Disprove the following statement.
 'If $p > q$, then $\frac{1}{p} < \frac{1}{q}$.' [2]
 (ii) State a condition on p and q so that the statement is true. [1]
- 7 The variables x and y satisfy the equation $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 5$.
 (i) Show that $\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$. [4]
 Both x and y are functions of t .
 (ii) Find the value of $\frac{dy}{dt}$ when $x = 1$, $y = 8$ and $\frac{dx}{dt} = 6$. [3]

Section B (36 marks)

- 8 Fig. 8 shows the curve $y = x^2 - \frac{1}{8} \ln x$. P is the point on this curve with x -coordinate 1, and R is the point $(0, -\frac{7}{8})$.

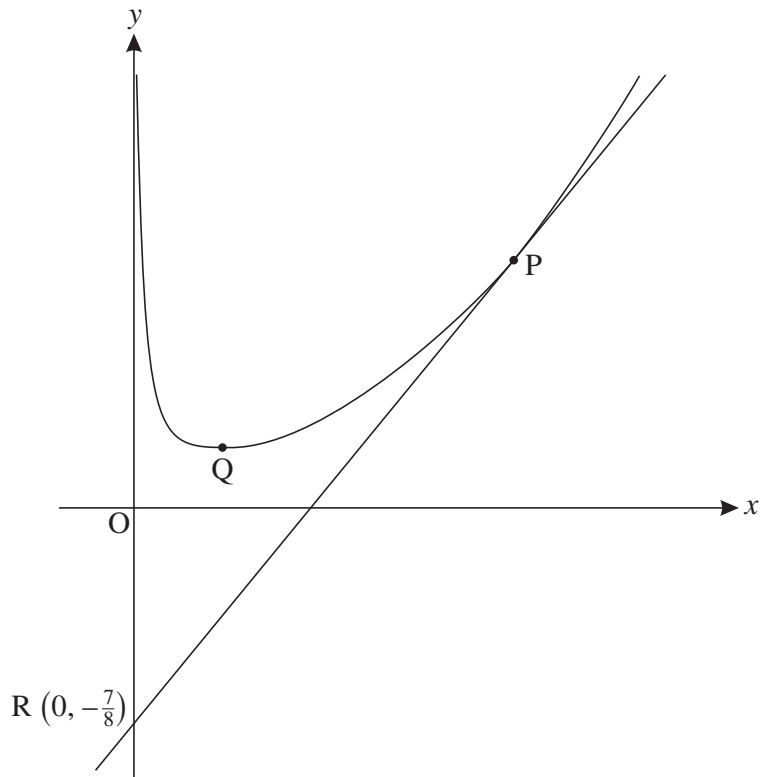


Fig. 8

- (i) Find the gradient of PR. [3]
- (ii) Find $\frac{dy}{dx}$. Hence show that PR is a tangent to the curve. [3]
- (iii) Find the exact coordinates of the turning point Q. [5]
- (iv) Differentiate $x \ln x - x$.

Hence, or otherwise, show that the area of the region enclosed by the curve $y = x^2 - \frac{1}{8} \ln x$, the x -axis and the lines $x = 1$ and $x = 2$ is $\frac{59}{24} - \frac{1}{4} \ln 2$. [7]

[Question 9 is printed overleaf.]

- 9 Fig. 9 shows the curve $y = f(x)$, where $f(x) = \frac{1}{\sqrt{2x - x^2}}$.

The curve has asymptotes $x = 0$ and $x = a$.

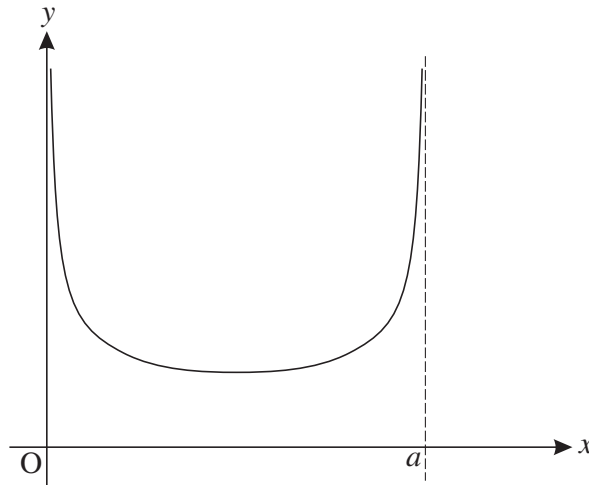


Fig. 9

- (i) Find a . Hence write down the domain of the function. [3]

(ii) Show that $\frac{dy}{dx} = \frac{x - 1}{(2x - x^2)^{\frac{3}{2}}}$.

Hence find the coordinates of the turning point of the curve, and write down the range of the function. [8]

The function $g(x)$ is defined by $g(x) = \frac{1}{\sqrt{1 - x^2}}$.

- (iii) (A) Show algebraically that $g(x)$ is an even function.
 (B) Show that $g(x - 1) = f(x)$.
 (C) Hence prove that the curve $y = f(x)$ is symmetrical, and state its line of symmetry. [7]