4754 (C4) Applications of Advanced Mathematics

Section A

1 $\frac{3x+2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{(x^2+1)}$ $\Rightarrow$ $3x + 2 = A(x^2 + 1) + (Bx + C)x$ $x = 0 \Rightarrow 2 = A$ coefft of x^2 : $0 = A + B \Rightarrow B = -2$ coefft of x: $3 = C$ $\Rightarrow$ $\frac{3x+2}{x(x^2+1)} = \frac{2}{x} + \frac{3-2x}{(x^2+1)}$	M1 M1 B1 M1 A1 A1 [6]	correct partial fractions equating coefficients at least one of B,C correct
2(i) $(1+2x)^{1/3} = 1 + \frac{1}{3}.2x + \frac{\frac{1}{3}.(-\frac{2}{3})}{2!}(2x)^2 + ...$ $= 1 + \frac{2}{3}x - \frac{2}{18}4x^2 + ...$ $= 1 + \frac{2}{3}x - \frac{4}{9}x^2 + ...$ * Next term $= \frac{\frac{1}{3}.(-\frac{2}{3})(-\frac{5}{3})}{3!}(2x)^3$ $= \frac{40}{81}x^3$ Valid for $-1 < 2x < 1$ $\Rightarrow -½ < x < ½$	M1 A1 E1 M1 A1 B1 [6]	binomial expansion correct unsimplified expression simplification www
3 $4\mathbf{j} - 3\mathbf{k} = \lambda\,\mathbf{a} + \mu\,\mathbf{b}$ $= \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k}) + \mu(4\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ $\Rightarrow$ $0 = 2\lambda + 4\mu$ $4 = \lambda - 2\mu$ $-3 = -\lambda + \mu$ $\Rightarrow$ $\lambda = -2\mu$, $2\lambda = 4 \Rightarrow \lambda = 2$, $\mu = -1$	M1 M1 A1 A1, A1 [5]	 equating components at least two correct equations
4 LHS = $\cot\beta - \cot\alpha$ $= \frac{\cos\beta}{\sin\beta} - \frac{\cos\alpha}{\sin\alpha}$ $= \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\sin\alpha\sin\beta}$ $= \frac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta}$ **OR** RHS= $\frac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta} = \frac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta} - \frac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta}$ $= \cot\beta - \cot\alpha$	M1 M1 E1 M1 M1 E1 [3]	cot = cos / sin combining fractions www using compound angle formula splitting fractions using cot=cos/sin

5(i) Normal vectors $\begin{pmatrix}2\\-1\\1\end{pmatrix}$ and $\begin{pmatrix}1\\0\\-1\end{pmatrix}$ Angle between planes is θ, where $\cos\theta = \dfrac{2\times1+(-1)\times0+1\times(-1)}{\sqrt{2^2+(-1)^2+1^2}\sqrt{1^2+0^2+(-1)^2}}$ $= 1/\sqrt{12}$ $\Rightarrow \theta = 73.2°$ or 1.28 rads	B1 M1 M1 A1 [4]	 scalar product finding invcos of scalar product divided by two modulae
(ii) $\mathbf{r} = \begin{pmatrix}2\\0\\1\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\1\end{pmatrix}$ $= \begin{pmatrix}2+2\lambda\\-\lambda\\1+\lambda\end{pmatrix}$ $\Rightarrow 2(2 + 2\lambda) - (-\lambda) + (1 + \lambda) = 2$ $\Rightarrow 5 + 6\lambda = 2$ $\Rightarrow \lambda = -½$ So point of intersection is (1, ½ , ½)	B1 M1 A1 A1 [4]	
6(i) $\cos\theta + \sqrt{3}\sin\theta = r\cos(\theta - \alpha)$ $= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ $\Rightarrow R\cos\alpha = 1,\ R\sin\alpha = \sqrt{3}$ $\Rightarrow R^2 = 1^2 + (\sqrt{3})^2 = 4, R = 2$ $\tan\alpha = \sqrt{3}$ $\Rightarrow \alpha = \pi/3$	B1 M1 M1 A1 [4]	$R = 2$ equating correct pairs $\tan\alpha = \sqrt{3}$ o.e.
(ii) derivative of $\tan\theta$ is $\sec^2\theta$ $\int_0^{\frac{\pi}{3}} \dfrac{1}{(\cos\theta+\sqrt{3}\sin\theta)^2}d\theta = \int_0^{\frac{\pi}{3}} \dfrac{1}{4}\sec^2(\theta-\dfrac{\pi}{3})d\theta$ $= \left[\dfrac{1}{4}\tan(\theta-\dfrac{\pi}{3})\right]_0^{\frac{\pi}{3}}$ $= ¼ (0 - (-\sqrt{3}))$ $= \sqrt{3}/4$ *	B1 M1 A1 E1 [4]	 ft their α $\dfrac{1}{R^2}$ [tan ($\theta - \pi/3$)] ft their R,α(in radians) www

Section B

7(i) (*A*) 9 / 1.5 = 6 hours (*B*) 18/1.5 = 12 hours	B1 B1 [2]	
(ii) $\frac{d\theta}{dt} = -k(\theta - \theta_0)$ $\Rightarrow \int \frac{d\theta}{\theta - \theta_0} = \int -kdt$ $\Rightarrow \ln(\theta - \theta_0) = -kt + c$ $\theta - \theta_0 = e^{-kt+c}$ $\theta = \theta_0 + Ae^{-kt}$ *	 M1 A1 A1 M1 E1 [5]	 separating variables $\ln(\theta - \theta_0)$ $-kt + c$ anti-logging correctly(with c) $A = e^{c}$
(iii) 98 = 50 + Ae^0 $\Rightarrow$ A = 48 Initially $\frac{d\theta}{dt} = -k(98-50) = -48k = -1.5$ $\Rightarrow$ k = 0.03125*	M1 A1 M1 E1 [4]	
(iv) (*A*) $89 = 50 + 48e^{-0.03125t}$ $\Rightarrow$ 39/48 = $e^{-0.03125t}$ $\Rightarrow$ t = ln(39/48)/(–0.03125) = 6.64 hours (*B*) $80 = 50 + 48e^{-0.03125t}$ $\Rightarrow$ 30/48 = $e^{-0.03125t}$ $\Rightarrow$ t = ln(30/48)/(–0.03125) = 15 hours	M1 M1 A1 M1 A1 [5]	equating taking lns correctly for either
(v) Models disagree more for greater temperature loss	B1 [1]	

8(i) $\frac{dy}{d\theta} = 2\cos 2\theta - 2\sin\theta$, $\frac{dx}{d\theta} = 2\cos\theta$ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\cos 2\theta - 2\sin\theta}{2\cos\theta} = \frac{\cos 2\theta - \sin\theta}{\cos\theta}$	B1, B1 M1 A1 [4]	 substituting for theirs oe
(ii) When $\theta = \pi/6$, $\frac{dy}{dx} = \frac{\cos\pi/3 - \sin\pi/6}{\cos\pi/6}$ $= \frac{1/2 - 1/2}{\sqrt{3}/2} = 0$ Coords of B: $x = 2 + 2\sin(\pi/6) = 3$ $y = 2\cos(\pi/6) + \sin(\pi/3) = 3\sqrt{3}/2$ BC $= 2\times 3\sqrt{3}/2 = 3\sqrt{3}$	E1 M1 A1,A1 B1ft [5]	 for either exact
(iii) (*A*) $y = 2\cos\theta + \sin 2\theta$ $= 2\cos\theta + 2\sin\theta\cos\theta$ $= 2\cos\theta(1 + \sin\theta)$ $= x\cos\theta$ * (*B*) $\sin\theta = ½ (x - 2)$ $\cos^2\theta = 1 - \sin^2\theta$ $= 1 - ¼ (x - 2)^2$ $= 1 - ¼ x^2 + x - 1$ $= (x - ¼ x^2)$ * (*C*) Cartesian equation is $y^2 = x^2\cos^2\theta$ $= x^2(x - ¼ x^2)$ $= x^3 - ¼ x^4$*	M1 E1 B1 M1 E1 M1 E1 [7]	$\sin 2\theta = 2\sin\theta\cos\theta$ squaring and substituting for x
(iv) $V = \int_0^4 \pi y^2 dx$ $= \pi\int_0^4 (x^3 - \frac{1}{4}x^4)dx$ $= \pi\left[\frac{1}{4}x^4 - \frac{1}{20}x^5\right]_0^4$ $= \pi(64 - 51.2)$ $= 12.8\pi = 40.2$ (m^3)	M1 B1 A1 [3]	need limits $\left[\frac{1}{4}x^4 - \frac{1}{20}x^5\right]$ 12.8π or 40 or better.

Comprehension

1	$\frac{400\pi d}{1000}=10$ $d=\frac{25}{\pi}=7.96$	M1 E1	
2	$V=\ \pi 20^2 h+\frac{1}{2}\left(\pi 20^2 H-\pi 20^2 h\right)$ $=\frac{1}{2}\left(\pi 20^2 H+\pi 20^2 h\right)\text{cm}^3 = 200\pi\left(H+h\right)\ \text{cm}^3$ $=\frac{1}{5}\pi\left(H+h\right)$ litres	M1 M1 E1	divide by 1000
3	$H=5+40\tan 30^\circ$ or $H=h+40\tan\theta$ $V=\frac{1}{5}\pi\left(H+h\right)=\frac{1}{5}\pi\left(10+40\tan 30^\circ\right)$ =20.8 litres	B1 M1 A1	or evaluated including substitution of values
4	$V=\frac{1}{2}\times 80\times\left(40+5\right)$ $\times 30\ \text{cm}^3 = 54\ 000\text{cm}^3$ = 54 litres	M1 M1 A1	×30
5	(i) Accurate algebraic simplification to give $y^2-160y+400=0$ (ii) Use of quadratic formula (or other method) to find other root: $d=157.5$ cm. This is greater than the height of the tank so not possible	B1 M1 A1 E1	
6	$y=10$ Substitute for y in (4): $V=\frac{1}{1000}\int_0^{100}375\text{d}x$ $V=\frac{1}{1000}\times 37500=37.5$ *	B1 M1 E1 [18]	