OCR
Oxford Cambridge and RSA

Thursday 21 October 2021 – Afternoon

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Time allowed: 1 hour 15 minutes

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You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

A surface S is defined by $z = \mathrm{f}(x, y)$ where $\mathrm{f}(x, y) = x^3 + x^2y - 2y^2$.

(a) On the coordinate axes in the Printed Answer Booklet, sketch the section $z = \mathrm{f}(2, y)$ giving the coordinates of any turning points and any points of intersection with the axes. **[4]**

(b) Find the stationary points on S. **[7]**

2 G is a group of order 8.

(a) Explain why there is no subgroup of G of order 6. **[1]**

You are now given that G is a cyclic group with the following features:

- e is the identity element of G,
- g is a generator of G,
- H is the subgroup of G of order 4.

(b) Write down the possible generators of H. **[2]**

M is the group $(\{0, 1, 2, 3, 4, 5, 6, 7\}, +_8)$ where $+_8$ denotes the binary operation of addition modulo 8. You are given that M is isomorphic to G.

(c) Specify all possible isomorphisms between M and G. **[4]**

3 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 3 & 3 & 0 \\ 0 & 2 & 2 \\ 1 & 3 & 4 \end{pmatrix}$.

(a) Determine the characteristic equation of $\mathbf{A}$. **[3]**

(b) Hence verify that the eigenvalues of $\mathbf{A}$ are 1, 2 and 6. **[1]**

(c) For each eigenvalue of $\mathbf{A}$ determine an associated eigenvector. **[4]**

(d) Use the results of parts **(b)** and **(c)** to find $\mathbf{A}^n$ as a single matrix, where n is a positive integer. **[6]**

4 The sequence $u_0, u_1, u_2, \ldots$ satisfies the recurrence relation $u_{n+2} - 3u_{n+1} - 10u_n = 24n - 10$.

(a) Determine the general solution of the recurrence relation. **[6]**

(b) Hence determine the particular solution of the recurrence relation for which $u_0 = 6$ and $u_1 = 10$. **[3]**

(c) Show, by direct calculation, that your solution in part **(b)** gives the correct value for u_2. **[1]**

The sequence $v_0, v_1, v_2, \ldots$ is defined by $v_n = \frac{u_n}{p^n}$ for some constant p, where u_n denotes the particular solution found in part **(b)**.

You are given that v_n converges to a finite non-zero limit, q, as $n \to \infty$.

(d) Determine p and q. **[4]**

5 A surface S is defined for $z \geqslant 0$ by $x^2 + y^2 + 2z^2 = 126$. C is the set of points on S for which the tangent plane to S at that point intersects the x-y plane at an angle of $\frac{1}{3}\pi$ radians.

Show that C lies in a plane, Π, whose equation should be determined. **[6]**

6 You are given that $q \in \mathbb{Z}$ with $q \geqslant 1$ and that

$$S = \frac{1}{(q+1)} + \frac{1}{(q+1)(q+2)} + \frac{1}{(q+1)(q+2)(q+3)} + \ldots.$$

(a) By considering a suitable geometric series show that $S < \frac{1}{q}$. **[3]**

(b) Deduce that $S \notin \mathbb{Z}$. **[2]**

You are also given that $\mathrm{e} = \sum_{r=0}^{\infty} \frac{1}{r!}$.

(c) Assume that $\mathrm{e} = \frac{p}{q}$, where p and q are positive integers. By writing the infinite series for e in a form using q and S and using the result from part **(b)**, prove by contradiction that e is irrational. **[3]**

END OF QUESTION PAPER