



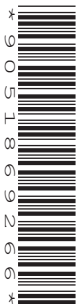
Oxford Cambridge and RSA

# Thursday 21 October 2021 – Afternoon

## A Level Further Mathematics B (MEI)

**Y435/01** Extra Pure

**Time allowed: 1 hour 15 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

### INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

### INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- This document has **4** pages.

### ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

**1 In this question you must show detailed reasoning.**

A surface  $S$  is defined by  $z = f(x, y)$  where  $f(x, y) = x^3 + x^2y - 2y^2$ .

- (a) On the coordinate axes in the Printed Answer Booklet, sketch the section  $z = f(2, y)$  giving the coordinates of any turning points and any points of intersection with the axes. [4]
- (b) Find the stationary points on  $S$ . [7]

**2**  $G$  is a group of order 8.

- (a) Explain why there is no subgroup of  $G$  of order 6. [1]

You are now given that  $G$  is a cyclic group with the following features:

- $e$  is the identity element of  $G$ ,
- $g$  is a generator of  $G$ ,
- $H$  is the subgroup of  $G$  of order 4.

- (b) Write down the possible generators of  $H$ . [2]

$M$  is the group  $(\{0, 1, 2, 3, 4, 5, 6, 7\}, +_8)$  where  $+_8$  denotes the binary operation of addition modulo 8. You are given that  $M$  is isomorphic to  $G$ .

- (c) Specify all possible isomorphisms between  $M$  and  $G$ . [4]

**3** The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} 3 & 3 & 0 \\ 0 & 2 & 2 \\ 1 & 3 & 4 \end{pmatrix}$ .

- (a) Determine the characteristic equation of  $\mathbf{A}$ . [3]
- (b) Hence verify that the eigenvalues of  $\mathbf{A}$  are 1, 2 and 6. [1]
- (c) For each eigenvalue of  $\mathbf{A}$  determine an associated eigenvector. [4]
- (d) Use the results of parts (b) and (c) to find  $\mathbf{A}^n$  as a single matrix, where  $n$  is a positive integer. [6]

4 The sequence  $u_0, u_1, u_2, \dots$  satisfies the recurrence relation  $u_{n+2} - 3u_{n+1} - 10u_n = 24n - 10$ .

(a) Determine the general solution of the recurrence relation. [6]

(b) Hence determine the particular solution of the recurrence relation for which  $u_0 = 6$  and  $u_1 = 10$ . [3]

(c) Show, by direct calculation, that your solution in part (b) gives the correct value for  $u_2$ . [1]

The sequence  $v_0, v_1, v_2, \dots$  is defined by  $v_n = \frac{u_n}{p^n}$  for some constant  $p$ , where  $u_n$  denotes the particular solution found in part (b).

You are given that  $v_n$  converges to a finite non-zero limit,  $q$ , as  $n \rightarrow \infty$ .

(d) Determine  $p$  and  $q$ . [4]

5 A surface  $S$  is defined for  $z \geq 0$  by  $x^2 + y^2 + 2z^2 = 126$ .  $C$  is the set of points on  $S$  for which the tangent plane to  $S$  at that point intersects the  $x$ - $y$  plane at an angle of  $\frac{1}{3}\pi$  radians.

Show that  $C$  lies in a plane,  $\Pi$ , whose equation should be determined. [6]

6 You are given that  $q \in \mathbb{Z}$  with  $q \geq 1$  and that

$$S = \frac{1}{(q+1)} + \frac{1}{(q+1)(q+2)} + \frac{1}{(q+1)(q+2)(q+3)} + \dots$$

(a) By considering a suitable geometric series show that  $S < \frac{1}{q}$ . [3]

(b) Deduce that  $S \notin \mathbb{Z}$ . [2]

You are also given that  $e = \sum_{r=0}^{\infty} \frac{1}{r!}$ .

(c) Assume that  $e = \frac{p}{q}$ , where  $p$  and  $q$  are positive integers. By writing the infinite series for  $e$  in a form using  $q$  and  $S$  and using the result from part (b), prove by contradiction that  $e$  is irrational. [3]

**END OF QUESTION PAPER**

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