GCE

Further Mathematics A

Y544/01: Discrete Mathematics

Advanced GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for A Level Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to 3 s.f. unless the question asks for a specific degree of accuracy.

Follow through should be used so that only one mark is lost for each distinct accuracy error.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th><th></th></tr>
<tr><td>1</td><td>(a)</td><td>(i)</td><td>A ↔ R
B ↔ Q
C ↔ S (or T)
D ↔ P
E ↔ T (or S)</td><td>B1

B1</td><td>2.1

2.1</td><td>A ↔ R, B ↔ Q, D ↔ P

C ↔ S and E ↔ T

(Or any three correct = B1 only)</td><td>May use any symbol to indicate correspondence

Or a reasoned argument or shown in a diagram</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>V + R = E + 2
5 + R = 6 + 2 ⇒ R = 3

<table><tr><td>Vertices</td><td>A, B, C, D, E</td><td>V = 5</td></tr><tr><td>Edges</td><td>AB, AC, AE, BC, BD, BE</td><td>E = 6</td></tr><tr><td>Regions</td><td>ABC, ABE, ACBE</td><td>R = 3</td></tr></table></td><td>B1

B1</td><td>1.2

2.1</td><td>Sub V = 5 and E = 6 into formula or 5 + 3 = 6 + 2 or equivalent
Describing the three regions by listing vertices or ‘ABC, ABE and outside region (infinite region)’ or equivalent</td><td>Need evidence of calculation not just R = 3
Vertices in any order (e.g. ABCE for ACBE) allow inclusion of D and repeats (e.g. ACBDBEA for ACBEA)</td></tr>
<tr><td></td><td>(b)</td><td></td><td>Make K_5 by adding arcs
AD, CD, CE, DE</td><td>M1
A1</td><td>2.4
1.1</td><td>K_5 or arc AD
All correct</td><td>Allow G2 used</td></tr>
<tr><td></td><td>(c)</td><td></td><td>Make $K_{3,3}$ (with an extra arc) by adding vertex U and arcs PR, PU, SU, TU</td><td>M1
A1</td><td>2.4
1.1</td><td>$K_{3,3}$ (as a subgraph) or arc PR
All correct</td><td>Allow G1 used</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[8]</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td>(a)</td><td></td><td><table><tr><td>Activity</td><td>Immediate predecessors</td></tr><tr><td>A</td><td>-</td></tr><tr><td>B</td><td>-</td></tr><tr><td>C</td><td>A</td></tr><tr><td>D</td><td>A, B</td></tr><tr><td>E</td><td>C, D</td></tr><tr><td>F</td><td>D</td></tr></table></td><td>B1
B1</td><td>1.1
1.1</td><td>Rows C and F correct
Rows D and E correct

With nothing in rows A and B</td><td></td></tr>
<tr><td></td><td>(b)</td><td></td><td><table><tr><td>Activity</td><td>Latest start time (days)</td></tr><tr><td>B</td><td>1</td></tr><tr><td>C</td><td>5</td></tr><tr><td>F</td><td>8</td></tr></table></td><td>B1</td><td>1.1</td><td>All correct</td><td></td></tr>
<tr><td></td><td>(c)</td><td></td><td>Construction</td><td>B1</td><td>1.2</td><td></td><td></td></tr>
</table>

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th colspan="2">Guidance</th></tr>
<tr><td></td><td>(d)</td><td>(i)</td><td>$^8C_2 = 28$ ways to choose 2 workers for A</td><td>B1</td><td>1.1</td><td>28</td><td></td></tr>
<tr><td></td><td></td><td>(ii)</td><td>Number of workers for B can be 1, 2 or 3
$^6C_1 + {}^6C_2 + {}^6C_3 = 6 + 15 + 20 = 41$</td><td>M1
A1</td><td>3.1a
1.1</td><td>1, 2, or 3 workers for B
41</td><td>Final answer 63 = SC1</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[7]</td><td></td><td></td><td></td></tr>
<tr><td>3</td><td>(a)</td><td></td><td>Maximise $P = 2x - z$
subject to $x + y + z \leq 60$
$2x + 3y + 4z \leq 60$
and $x \geq 0, y \geq 0, z \geq 0$</td><td>B1

M1
A1</td><td>3.1a

1.1
1.1</td><td>‘Max’ and objective function $2x - z$
or any non-negative multiple of this
Either constraint correct in this form
Both constraints <u>and</u> non-negativity
(trivial constraints) correct</td><td>‘Max’ and $P - 2x + z = 0$

Form $ax + by + cz \leq d$
May have non-negative
multiples of constraints</td></tr>
<tr><td></td><td>(b)</td><td></td><td><table><tr><td>P</td><td>x</td><td>y</td><td>z</td><td>s</td><td>t</td><td>RHS</td></tr><tr><td>1</td><td>-2</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>60</td></tr><tr><td>0</td><td>2</td><td>3</td><td>4</td><td>0</td><td>1</td><td>60</td></tr></table>Pivot on 2 in row 3 of column x

<table><tr><td>P</td><td>x</td><td>y</td><td>z</td><td>s</td><td>t</td><td>RHS</td></tr><tr><td>1</td><td>0</td><td>3</td><td>5</td><td>0</td><td>1</td><td>60</td></tr><tr><td>0</td><td>0</td><td>-0.5</td><td>-1</td><td>1</td><td>-0.5</td><td>30</td></tr><tr><td>0</td><td>1</td><td>1.5</td><td>2</td><td>0</td><td>0.5</td><td>30</td></tr></table></td><td>B1

M1
A1</td><td>1.1

1.1
1.1</td><td>May be implied from iterated tableau

Dealing with (their) pivot in 3rd row
All correct</td><td>$60 \div 2 = 30 < 60 \div 1$

Using decimals or fractions</td></tr>
<tr><td></td><td>(c)</td><td></td><td>$2x + 3y + 4z + t = 60 \Rightarrow x = 30 - 1.5y - 2z - 0.5t$
Substitute for x:
$P - 2x + z = 0$
$\Rightarrow P - (60 - 3y - 4z - t) + z = 0$
$\Rightarrow P + 3y + 5z + t = 60$
$x + y + z + s = 60$
$\Rightarrow (30 - 1.5y - 2z - 0.5t) + y + z + s = 60$
$\Rightarrow -0.5y - z + s - 0.5t = 30$</td><td>
M1

A1

A1</td><td>
2.1

2.2a

2.2a</td><td>Eliminate x by substitution
Showing substitution for x

$P + 3y + 5z + t = 60$ o.e.
from algebraic substitution seen
$-0.5y - z + s - 0.5t = 30$ o.e.
from algebraic substitution seen</td><td>Not showing calculation of each row as a linear combination of other rows

Not just algebraic interpretation of tableau</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[9]</td><td></td><td></td><td></td></tr>
</table>

Question			Answer	Marks	AO	Guidance	
4	(a)		To ensure that the algorithm is finite Or reference to using a stopping condition	**B1**	**1.1**	So it does not loop indefinitely Not 'count iterations' unless also refer to using a 'stopping condition'	Not ' because it is finite' Not measuring efficiency
	(b)		20 seconds	**B1**	**1.1**	$0.2 \times \left(\frac{5000}{500}\right)^2$	
	(c)		Practical problems are usually large and cannot be solved efficiently by ad hoc methods	**B1** **B1**	**1.1** **2.4**	Large problems Algorithms are more efficient May take a long time otherwise	 A computer would need precise instructions
	(d)		**41** 17 8 33 29 **17** 8 33 29 41 **8** 17 **33** 29 41 8 17 **29** 33 41 8 17 29 33 41	**M1** **A1** **A1** **A1**	**1.1** **1.1** **1.1** **1.1**	Pivot on first value First iteration correct Second iteration correct with 17, 41 indicated in some way as being fixed At least one more iteration to switch 33 and 29	Any notation used consistently
	(e)		Average case for bubble (or shuttle) sort is $O(n^2)$ $O(n \log n) \subset O(n^2)$	 **B1**	 **1.2**	 $O(n \log n) \subset O(n^2)$ or in words	
	(f)		<table><tr><th></th><th>Worst case</th><th>Comparisons</th></tr><tr><td>Quick sort</td><td>8 17 29 33 41</td><td>10</td></tr><tr><td>Bubble sort</td><td>41 33 29 17 8</td><td>10</td></tr></table> Both use 4 + 3 + 2 + 1 (= 10) comparisons	**M1** **M1** **A1**	**1.1** **1.1** **1.1**	Worst case for quick sort (pivot 1st) Worst case for bubble sort 10 or 4 + 3 + 2 + 1 seen, dep M1 M1	Increasing or decreasing order or any other worst case Decreasing order Allow any calculation that gives the answer 10
				[12]			

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th><th></th></tr>
<tr><td>5</td><td>(a)</td><td></td><td>Matrix is symmetric about lead diagonal</td><td>E1</td><td>2.5</td><td>Table is symmetric about diagonal
Matrix is its own transpose</td><td>Rows are same as columns
Examples and ‘always true’</td></tr>
<tr><td></td><td>(b)</td><td></td><td>A – C - E</td><td>B1</td><td>3.1a</td><td>ACE or AC, CE in any form</td><td></td></tr>
<tr><td></td><td>(c)</td><td></td><td><table><tr><th>Vertex</th><th>Temporary labels</th><th>Order of permanent labelling</th><th>Permanent label</th></tr><tr><td>A</td><td>300, 280</td><td>7</td><td>280</td></tr><tr><td>B</td><td>170</td><td>4</td><td>170</td></tr><tr><td>C</td><td>180</td><td>5</td><td>180</td></tr><tr><td>D</td><td>120</td><td>3</td><td>120</td></tr><tr><td>E</td><td>210</td><td>6</td><td>210</td></tr><tr><td>F</td><td>350</td><td>8</td><td>350</td></tr><tr><td>G</td><td></td><td>1</td><td>0</td></tr><tr><td>H</td><td>90</td><td>2</td><td>90</td></tr></table></td><td>

M1

M1

A1

B1ft</td><td>

1.1

1.1

1.1

1.1</td><td>Working may be done on a network.

Temp labels 170 at B, 120 at D and 90 at H
Updating at A

All permanent labels correct and no extra temp labels
Order of labelling correct for their permanent labels</td><td>

Dependent on both M marks

From 1 to 8</td></tr>
<tr><td></td><td>(d)</td><td></td><td>2 × length of all roads = 3220 metres</td><td>B1</td><td>3.2a</td><td>3220m or 3.22 km, with units</td><td></td></tr>
<tr><td></td><td>(e)</td><td>(i)</td><td>AE = 180 AF = 250 AG = 280
FG = <u>350</u> EG = <u>210</u> EF = <u>140</u>
530 460 420
Repeat ACHG and EF
420 + 1610
= 2030 metres</td><td>M1
B1
A1

M1
A1</td><td>3.3
1.1
3.4

3.4
3.5a</td><td>Considering these three <u>pairs</u>
AE = 180 seen
530, 460 and 420 seen

Their 420 + [1610 or from their (ii)]
2030m or 2.03 km, with units</td><td>
AC, CE = 180

Addition seen (or implied form correct answer)</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td>SC if a candidate gives 1880 and explains that they have doubled the shortest route from B to an odd vertex (BA = 130) and EF (= 140)</td><td></td></tr>
<tr><td></td><td></td><td>(ii)</td><td>Length = 1750 m
Start at A</td><td>B1
B1</td><td>3.4
3.1b</td><td>1750 or 1.75 as shortest length
Or start at G</td><td>
A or G (or both) as start</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[14]</td><td></td><td></td><td></td></tr>
</table>

Question			Answer	Marks	AO	Guidance	
6	(a)		$x \geq 3$	B1	2.1	Allow $x > 3$	
	(b)		If Vlad plays X, Sumi's highest score is by playing A If Sumi plays A, Vlad's highest score is by playing X	E1 E1	2.4 2.4	Max$\{x, 3\} = x$ Max $\{1, -2, 0\} = 1$	Two separate statements, not merged into one
	(iii)		<table><tr><td></td><td>X</td><td>Y</td><td>Z</td><td>Min pay-off Sumi</td></tr><tr><td>A</td><td>(x, 1)</td><td>(4, -2)</td><td>(2, 0)</td><td>2 *</td></tr><tr><td>B</td><td>(3, -1)</td><td>(6, -4)</td><td>(-1, 3)</td><td>-1</td></tr><tr><td>Min pay-off Vlad</td><td>-1</td><td>-4</td><td>0 *</td><td></td></tr></table> Play-safe for Sumi is A, maximin pay-off = 2 Play-safe for Vlad is Z, maximin pay-off = 0. Maximin pay-off for Sumi is 2 and maximin pay-off for Vlad is 0. Cell (A, Z) has pay-off 2 for Sumi and pay-off 0 for Vlad.	 M1 M1 A1	 1.1 1.1 1.1	 Finding play-safe or maximin Sumi Finding play-safe or maximin Vlad (A, Z) = (2, 0)	 A or 2, -1 (allow '2 or x', 1) Z or -1, -4, 0 or 1, 4, 0 Cell (A, Z) as well as 2 and 0 from correct working

Question			Answer	Marks	AO	Guidance	
	(d)		$x = 1$ |	X	Y	Z	row min |	**B1**	**3.1a**	Seen or implied from zero-sum pay-off = 0	Each cell must have the same sum
			A | 0 | 3 | 1 | 0 |	**M1**	**3.1a**	$2 \div 2 = 1$, subtract 1 from each score to get pay-off for Sumi (and negative of pay-off for Vladimir)	These entries for cells apart from (A, X), or a non-zero multiple
			B | 2 | 5 | -2 | -2 |	**A1**	**2.4**	Pay-off's for Sumi correct, or a positive multiple	With 0 for (A, X)
			col max | 2 | 5 | 1 |	**B1ft**	**2.4**	Verifying that game is unstable	
			Game is unstable ($0 \neq 1$) Sumi chooses randomly, P(A) = p Vlad plays X: $0(p) + 2(1 - p)$ or $2 - 2p$ (Vlad plays Y: $3(p) + 5(1 - p)$ or $5 - 2p$) Vlad plays X: $1(p) - 2(1 - p)$ or $3p - 2$	**M1ft**	**1.1**	Expected winnings for Sumi when Vlad plays X and Z in terms of one parameter (may still have constant x)	(Y is dominated by X so may be excluded, or not)
			$2 - 2p = 3p - 2$ $p = 0.8$ Choose randomly between rows, so that A is played with probability 0.8 and B with probability 0.2	**depM1** **A1**	**1.1** **3.2a**	Solving their X = their Z, algebraically, or using a graph $p = 0.8$, interpreted <u>in context</u>, following correct working	Allow their Y = their X or Z or max of their lower boundary (shown) Or interpret for B with 0.2
				[13]			

Question			Answer	Marks	AO	Guidance	
7	(a)		x = number of large pies made y = number of medium pies made z = number of small pies made	**B1** **B1**	**3.3** **1.1**	Number of (pies made) x = large, y = medium, z = small	
	(b)		$36x + 15y + 10z \leq 180$	**M1** **A1**	**3.3** **1.1**	Coefficients in ratio $\frac{1}{5} : \frac{1}{12} : \frac{1}{18}$ 36, 15, 10, 180 or any positive integer multiple of this set	Follow through their definitions of x, y, z if appropriate
	(c)		$z = 9 \Rightarrow 180x + 63y \leq 945 \Rightarrow 20x + 7y \leq 105$ $36x + 15y \leq 90 \Rightarrow 12x + 5y \leq 30$ and $x + y = 9$ $\Rightarrow 5x + 5y = 45$ so $12x + 5y$ cannot be ≤ 30	 **E1** **E1**	 **3.5c** **2.1**	 May be argued in words, algebraically or graphically Filling constraint cannot be satisfied	 Eliminate one variable to get two inequalities and total Or using one inequality to show that a variable is negative, or equivalent contradiction

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th colspan="2">Guidance</th></tr>
<tr><td></td><td>(d)</td><td></td><td>There is enough filling for 5 large pies, so $x \leq 5$</td><td></td><td></td><td>Upper limit 5</td><td>May be argued in words, algebraically or graphically</td></tr>
<tr><td></td><td></td><td></td><td>$180x + 63y + 35z \leq 1260$
$36x + 15y + 10z \leq 180$
$x + y = z$</td><td>B1</td><td>3.5c</td><td>Refining the model to take the new constraint into account</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td>Or branch-and-bound</td><td>Or ‘make no medium so $y = 0$ and $x = z$‘</td></tr>
<tr><td></td><td></td><td></td><td>Eliminating z:</td><td>M1</td><td>3.3</td><td>Or simplex with obj (max x) and at least 3 constraints</td><td>Or checking at least 3 feasible solutions</td></tr>
<tr><td></td><td></td><td></td><td>$215x + 98y \leq 1260$ and $46x + 25y \leq 180$</td><td>A1</td><td>2.4</td><td>Eliminating a variable
$(-117y+215z \leq 1260, -21y+46z \leq 180)$
$(117x+98z \leq 1260, 21x+25z \leq 180$
Or solve simplex with 4 correct constraints</td><td>Feasible cases are $(x, y, z) =$
$(0, k, k)$ for $k = 0, 1, \ldots, 6$
$(1, k, k+1)$ for $k = 0, 1, \ldots, 5$
$(2, k, k+2)$ for $k = 0, 1, 2, 3$
$(3, k, k+3)$ for $k = 0, 1$</td></tr>
<tr><td></td><td></td><td></td><td>$180 \div 46 = 3.913$, so $x \leq 3$</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>$x = 3 \Rightarrow 25y \leq 42 \Rightarrow y = 0$, $z = 3$ or $y = 1$, $z = 4$</td><td>M1</td><td>3.4</td><td>Either of these integer solutions</td><td></td></tr>
<tr><td></td><td></td><td></td><td>Verify that their solution satisfies the constraints
<table><tr><th></th><th>$180x+63y+35z$</th><th>$36x+15y+10z$</th></tr><tr><td>$x=3, y=0, z=3$</td><td>645</td><td>138</td></tr><tr><td>$x=3, y=1, z=4$</td><td>743</td><td>163</td></tr><tr><td>Upper limit</td><td>1260</td><td>180</td></tr></table></td><td>A1</td><td>2.2a</td><td>Shown to be consistent with all constraints (or implied from algebra)</td><td></td></tr>
<tr><td></td><td></td><td></td><td>The maximum number of large pies is 3</td><td>B1</td><td>3.4</td><td>Maximum is 3</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[12]</td><td></td><td></td><td></td></tr>
</table>

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