Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4761**: Mechanics 1

Advanced Subsidiary GCE

Mark Scheme for June 2017

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper
Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper
Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct error made in the accuracy to which working is done or an answer given. Refer cases to your Team Leader where the same type of error (e.g. errors due to premature approximation leading to error) has been made in different questions or parts of questions.

There are some mistakes that might be repeated throughout a paper. If a candidate makes such a mistake, (eg uses a calculator in wrong angle mode) then you will need to check the candidate’s script for repetitions of the mistake and consult your Team Leader about what penalty should be given.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

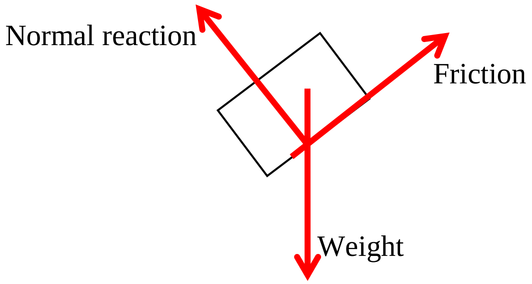
‘Fresh starts’ will not affect an earlier decision about a misread.

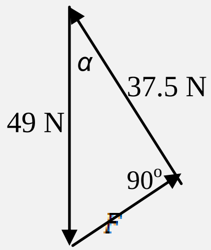
Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

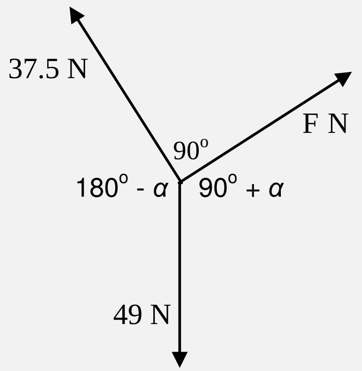
i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

SECTION A

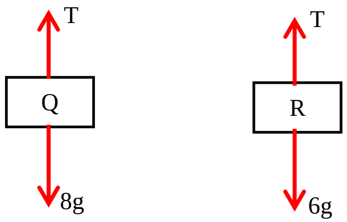
Qu	Part	Answer	Mark	Guidance
1	(i)		B1 B1 B1	**Forces** 3 correct forces. **Labels** Accept 5g and mg for weight. **Arrows** **Missing force(s)** If at least one of the 3 forces is missing, allow **SC1** for each fully correct force (ie including label and arrow) and ignore any additional forces that may be present. **Extra force(s)** Allow **B0** for **Forces** and up to **B1** for each of **Labels** and **Arrows** based on the correct forces and ignoring any extra(s). **Components of weight** Allow weight resolved into components parallel and perpendicular to the slope Accept **both** the weight and its components if the components are shown to be clearly different from the other forces (eg drawn with broken lines). Do not accept both the weight and its components if they all look the same; mark this as detailed under **Extra force(s)**.
			[3]	

1	**(ii)**	$N = 5g\cos\alpha$ (or 37.5 = 5g cos α) $\cos\alpha = \frac{37.5}{49}$ $\alpha = 40.065...°$ so $40°$ to the nearest degree Frictional force = component of weight down the slope $= 5g\sin 40.065...°$ $(= 31.539...)$ so 31.5 N	**M1** **A1** **B1**	Do not allow sin-cos interchange Must be rounded to 40^{o} Allow any answer that rounds to 31.5 N Allow answer 31.5 N following two consistent sin-cos interchanges.
			[3]	
		Alternative Using a triangle of forces  $\cos\alpha = \frac{37.5}{49}$ $\Rightarrow$ $\alpha = 40°$ $= 5g\sin 40.065...°$ $(= 31.539...)$ so 31.5 N	**M1** **A1** **B1**	Condone no arrows. Do not allow sin-cos interchange Must be rounded to 40^{o} Allow any answer that rounds to 31.5 N Allow answer 31.5 N following two consistent sin-cos interchanges.

1.	(ii)	**Alternative Using Lami's theorem** 		
		$\frac{\text{F}}{\sin(180°-\alpha)}=\frac{37.5}{\sin(90°+\alpha)}=\frac{49}{\sin 90°}$	**M1**	
		$\alpha = 40°$	**A1**	Must be rounded to 40°
		$\text{F} = 31.5\text{ N}$	**B1**	Allow any answer that rounds to 31.5 N

2	(i)	Differentiating **r**	M1	Attempt at differentiation must be seen
		v= $\begin{pmatrix} 2 \\ -4 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} t$	A1	
		v= $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$	B1	Apply ISW for speed = $\sqrt{5}$ providing $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is seen.
			[3]	
	(ii)	$\sqrt{2^2 + (-4+2t)^2} = 10$	M1	Attempt at formulation of the given information using their vector **v** from part (i). Must involve both components. e.g. $-4+2t=\sqrt{96}$ Accept drawing triangle of velocities
		$t^2 - 4t - 20 = 0$		
		$t = \dfrac{4 \pm \sqrt{4^2 - 4 \times 1 \times -20}}{2}$ $(= 6.898\ldots \text{ or } -2.898\ldots)$	M1	Attempted solution of an equation for t. Dependent on previous M mark
		$t = 6.9$ $(\text{or } -2.9)$ (to 2 sf)	A1	Allow FT from their vector expression for **v** in part (i). Else CAO. Condone not giving the negative value of t as well as the correct value. Dependent on both M marks.
			[3]	
	(iii)	Either 2 = -4 +2t $\Rightarrow t = 3$	B1	FT from their vector expression for **v** in part (i).
		Or $-2 = -4 + 2t \Rightarrow t = 1$	B1	FT from their vector expression for **v** in part (i).
			[2]	

3	(i)	$2 \times 120 \times \cos 20° - F = 430 \times 0.05$	M1	Newton's 2nd law, including ma term, friction and resolved force(s); allow sin-cos interchange for this mark only.
			A1	All terms and signs correct
		F = 204(.026 ...)	A1	
			[3]	
	(ii)	$430 \times a = 240 - 204.026...$	M1	Apply FT from their F from part (i) throughout this part. All forces present
		$a = 0.08366...$	A1	Condone 0.08 for this mark
		Percentage increase is $\frac{0.0836... - 0.05}{0.05} \times 100 \quad (= 67.32...)$	M1	There must be evidence of a complete method for finding percentage change. The denominator must be the original acceleration and the original value must be subtracted from the new value at some stage.
		Percentage increase is 67.3% (to nearest 0.1)	A1	To allow for rounding and truncation, allow answers between 66% and 68% inclusive following otherwise correct working.
			[4]	

4	(i)		B1	The same symbol for T must be used in both diagrams.
			[1]	
	(ii)	Q: $8g - T = 8a$ R: $T - 6g = 6a$	B1 B1	Allow the equivalent equations with the direction of a reversed
			[2]	
	(iii)	Adding the equations of motion $2g = 14a$	M1	Eliminating one variable from the two equations. May be implied by subsequent working.
		$a = \frac{2g}{14}$ $\left(= 1.4 \text{ m s}^{-2}\right)$	A1	This answer must be consistent with the direction of a used in part (ii)
		For Q: $s = ut + \frac{1}{2}at^2$		Or an equivalent sequence of constant acceleration formulae
		$2 = \frac{1}{2} \times 1.4 \times t^2$	M1	Dependent on previous M mark. FT for their a but do not allow if it is g
		$\Rightarrow t = 1.690...$ so the time is 1.69 s	A1	CAO
			[4]	

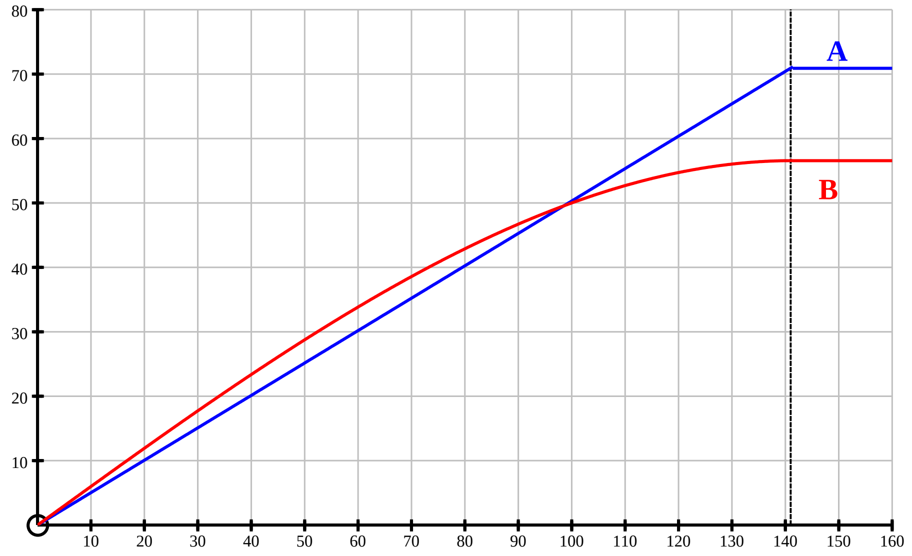
5	(i)	A: $x = t^2$	B1	Or displacements from B's start point: $x = t^2 + 75$ and $x = 20t$
		B: $x = -75 + 20t$	B1	Must be consistent
		When the cars are side by side, $t^2 = -75 + 20t$	M1	For equating two distances even if inconsistent
		$t^2 - 20t + 75 = 0$		
		$(t-5)(t-15) = 0$		
		The times are 5 seconds and 15 seconds	A1	
			[4]	
	(ii)	For A, s = 225 when t = 15		FT for two positive times from part (i) for the M mark only
		s = 25 when t = 5	M1	Both values of s attempted
		So A is behind B for 200 m	A1	CAO
			[2]	
		Alternative Using motion of B		
		Speed of B is (constant at) 20 m s^{-1}		
		So between t = 5 and t = 15, B travels 20 × (15- 5) (= 200 m)	M1	Or equivalent, eg using $s = ut + \frac{1}{2}at^2$ with a = 0
		So A is behind B for 200 m	A1	

5.	(ii)	**Alternative Using motion of A with the clock re-set** When the cars are first level, the motion of A is defined by $u = 10$ and $a = 2$. If the clock is re-set at this moment, $t = 0$ In this case, when they are next level, $t = 10$ $s = ut + \frac{1}{2}at^2 \Rightarrow s = 10\times10 + \frac{1}{2}\times2\times10^2$ $\Rightarrow s = 200$	M1 A1	Or $v = u + at \Rightarrow v = 10 + 2\times10 = 30$ followed by use of $v^2 - u^2 = 2as$
5	(iii)	For A $v^2 - u^2 = 2as$ $v^2 = 2\times2\times400$ $v = 40$, (so speed 40 $m\,s^{-1}$).	M1 A1	There must be an attempt to use the formula
			[2]	
		Alternative finding the time first For A $s = 400 \Rightarrow t = 20$ $v = u + at$ $\Rightarrow v = 40$ so 40 $m\,s^{-1}$	M1 A1	There must be evidence of a complete method that can lead to the value of v

SECTION B

6	(i)	2 minutes 21 seconds is 141 seconds	B1	
		$s = ut + \frac{1}{2}at^2$		
		$5000 = 0 + 0.5 \times a \times 141^2$	M1	
		$a = 0.503 \quad (\text{m s}^{-2})$	A1	Allow 0.50 but not 0.5
		$v = u + at$	M1	Or equivalent, eg $v^2 - u^2 = 2as$
		$v = 0.503 \times 141 = 70.9$ so 70.9 m s^{-1}	A1	CAO (including 70.5 m s^{-1})
			[5]	
		Alternative using $s = \frac{1}{2}(u+v)t$		
		$5000 = \frac{1}{2} \times (0 + v) \times 141$	M1	
		$v = \frac{10000}{141} = 70.9...$ so 70.9 m s^{-1}	A1	CAO

6	**(ii)**	At maximum speed the acceleration is zero $t=\sqrt{\frac{0.6}{3\times10^{-5}}}\ \left(=\sqrt{20\,000}\right)\ =141.421...$ So 2 minutes 21.42 seconds	**M1** **A1**	Setting a = 0 in the given equation for a. Accept answer in seconds
			[2]	
6	**(iii)**	Integrating $v=0.6t-0.000\,01t^3\ (+c)$, $(t=0,\ v=0\ \Rightarrow\ c=0)$ $s=0.3t^2-0.000\,0025t^4\ (+k)$ $t=0,\ s=0\ \Rightarrow\ k=0$	**M1** **A1** **A1** **A1**	Attempt at integration. Or equivalent, eg $v=0.6t-10^{-5}\times t^3\ (+c)$ Coefficients do not need to be simplified in either integral. FT from v. Integration must be attempted. Or equivalent, eg $s=0.3t^2-2.5\times10^{-6}\times t^4\ (+k)$ Use of mechanics and not assertion to show k = 0
			[4]	
	(iv)	Substituting t = 141.42... in $s=0.3t^2-0.000\,0025t^4$ $s=5000$ so consistent with 5 km Substituting t = 141.42... in $v=0.6t-0.000\,01t^3$ $v=56.57\ \mathrm{m\,s^{-1}}$	**M1** **A1** **B1**	Allow substituting s = 5000 to show that t =141.42... Notice that $141.42....=\sqrt{20\ 000}$ and so the answer of 5000 is exact
			[3]	

6	(v)		**B1** **B1** **B1** **B1**	Vertical scale from 0 that ensures that at least half the height is used. For the remaining marks do not allow FT from earlier parts. A: A straight line from (0, 0) to (141, 70.9) B: A curve from (0, 0) to (141, 56.6); the curvature must be in the right sense. Both A from (141, 70.9) to (160, 70.9) and B from (141, 56.6) to (160, 56.6). CAO.
			[4]	

7.	**(i)**	Initial speed = $\sqrt{10^2+5.5^2}=11.412...$ so 11.4 m s-1 (to 1 dp)	**B1**	
		$\alpha=\arctan\left(\frac{5.5}{10}\right)=28.810...$ so 28.8° (to the nearest 0.1°)	**B1**	
			[2]	
	(ii)	Horizontal motion: Time to net 10t = 12.5 so 1.25 s	**B1**	
		Vertical motion $s=s_0+ut+\frac{1}{2}at^2$		
		$y=2.22+5.5\times1.25-4.9\times1.25^2$	**M1**	A complete method for finding the height of the ball when it crosses the net. With the point of projection as the origin for vertical motion, the distance fallen in 1.25 s is 0.78125 m and 2.22 - 0.78125 =1.43875
		= 1.43875		
		This is greater than 0.995 so the ball goes over the net.	**A1**	Conclusion stated
			[3]	
		Alternative using time to net level and horizontal distance		
		$0.995=2.22+5.5t-4.9t^2 \Rightarrow t=1.313$	**B1**	
		Horizontal distance $10\times1.313=13.13$	**M1**	A complete method for finding the position of the ball when it is at the height of the top of the net.
		13.13 > 12.5 so the ball passes over the net	**A1**	Conclusion stated.

	(iii)	Vertical motion $s = s_0 + ut + \frac{1}{2}at^2$		
		$0 = 2.22 + 5.5t - 4.9t^2$	M1	Setting up an equation for vertical motion containing the right elements. (Vertical velocity on landing = 8.59 m s^{-1})
		$t = \frac{5.5 \pm \sqrt{5.5^2 + 4 \times 4.9 \times 2.22}}{2 \times 4.9} = 1.437...\ (\text{or } -0.315...)$	A1	
		Horizontal motion $x = 10 \times 1.437... = 14.37...$	M1	Allow for 10 × their time. This may be implied.
		14.37... < 19 so the ball does land in the service court	A1	Conclusion stated. FT for their value of t.
			[4]	

7	(iv)	**Clearing the net**		
		The ball falls 2.22 - 0.995 = 1.225 m to the height of the net	**M1**	
		Time taken is given by $1.225 = 4.9t^2$		
		So t = 0.5	**A1**	The value of t can be implied and need not be seen.
		Speed must be greater than $\frac{12.5}{0.5} = 25\text{ m s}^{-1}$	**A1**	
		Not going too far		
		Time to fall to the ground is given by $2.22 = 4.9t^2$	**M1**	
		So t = 0.673...	**A1**	The value of t can be implied and need not be seen.
		Horizontal distance must not exceed 19 m		
		Maximum speed = $\frac{19}{0.673...} = 28.227...\text{ m s}^{-1}$	**A1**	
		(Overall)		
		(So the ball's speed must be between 25 and 28.2 m s^{-1}.)		
			[6]	

7	(v)	Vertical motion $s = s_0 + ut + \frac{1}{2}at^2$		
		Using u to be the initial speed $0 = 2.22 - u \times \sin 2° \times 0.57 - 4.9 \times 0.57^2$	M1	An equation for vertical motion which could be used to find u. It must contain all three elements. No sin-cos interchange. If sin2° is not seen use the alternative method.
			A1	The equation must be correct including signs.
		$u = \dfrac{2.22 - 4.9 \times 0.57^2}{0.57 \times \sin 2°}$		
		$u = 31.568...$ so the speed of Tara's serve is 31.6 m s^{-1}	A1	CAO
			[3]	
		Alternative Using U as the initial vertical component downwards		Or equivalent for vertical motion upwards
		$0 = 2.22 - U \times 0.57 - 4.9 \times 0.57^2$	M1	
		$U = \dfrac{2.22 - 4.9 \times 0.57^2}{0.57} = 1.10173...$	A1	The value of U is calculated correctly. It should be negative if the direction of U is upwards.
		Speed = $\dfrac{U}{\sin 2°} = 31.568...$ So the speed of Tara's serve is 31.6 m s^{-1}	A1	

7. Alternative mark schemes for parts (ii), (iii) and (iv) using the equation of the trajectory

7.	**(ii)**	$y = y_0 + x\tan\alpha - \dfrac{gx^2}{2u^2\cos^2\alpha}$		
		$y = 2.22 + 0.55x - 0.049x^2$	**B1**	
		$x = 12.25$	**M1**	
		$\Rightarrow y = 1.438... > 0.995$	**A1**	
			[3]	
	(iii)	$y = 2.22 + 0.55x - 0.049x^2$	**M1**	
		$x = 19$ / Or $y = 0$	**M1**	
		$\Rightarrow y = -5.019$ / $x = 14.376...$ $(\text{or} -3.151...)$	**A1**	Dependent on both M marks
		So the ball lands in the service court	**A1**	
			[4]	

7	(iv)	$y = 2.22 - 0.049\left(\frac{x}{u}\right)^2$ **Clearing the net** To clear the net $2.22 - 4.9\left(\frac{12.5}{u}\right)^2 > 0.995$ $\Rightarrow \left(\frac{u}{12.5}\right)^2 > \left(\frac{4.9}{1.225}\right)$ Speed must be greater than $\frac{12.5}{0.5} = 25 \text{ m s}^{-1}$ **Not going too far** To land inside the service court, horizontal distance must not exceed 19 m $\Rightarrow 2.22 - 4.9\times\left(\frac{19}{u}\right)^2 < 0$ $\frac{u}{19} < \sqrt{\frac{4.9}{2.22}}$ $u < 28.227$ Maximum speed = $28.227... \text{ m s}^{-1}$ (So the ball's speed must be between 25 and 28.2 m s^{-1}.)	 M1 A1 B1 M1 A1 A1	
			[6]	

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