

General Certificate of Education  
January 2006  
Advanced Level Examination



**MATHEMATICS**  
**Unit Pure Core 4**

**MPC4**

Wednesday 25 January 2006 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
  - the **blue** AQA booklet of formulae and statistical tables
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

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1 (a) The polynomial  $f(x)$  is defined by  $f(x) = 3x^3 + 2x^2 - 7x + 2$ .

(i) Find  $f(1)$ . (1 mark)

(ii) Show that  $f(-2) = 0$ . (1 mark)

(iii) Hence, or otherwise, show that

$$\frac{(x-1)(x+2)}{3x^3 + 2x^2 - 7x + 2} = \frac{1}{ax+b}$$

where  $a$  and  $b$  are integers. (3 marks)

(b) The polynomial  $g(x)$  is defined by  $g(x) = 3x^3 + 2x^2 - 7x + d$ .

When  $g(x)$  is divided by  $(3x - 1)$ , the remainder is 2. Find the value of  $d$ . (3 marks)

2 A curve is defined by the parametric equations

$$x = 3 - 4t \quad y = 1 + \frac{2}{t}$$

(a) Find  $\frac{dy}{dx}$  in terms of  $t$ . (4 marks)

(b) Find the equation of the tangent to the curve at the point where  $t = 2$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers. (4 marks)

(c) Verify that the cartesian equation of the curve can be written as

$$(x-3)(y-1) + 8 = 0 \quad (3 \text{ marks})$$

3 It is given that  $3 \cos \theta - 2 \sin \theta = R \cos(\theta + \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ .

(a) Find the value of  $R$ . (1 mark)

(b) Show that  $\alpha \approx 33.7^\circ$ . (2 marks)

(c) Hence write down the maximum value of  $3 \cos \theta - 2 \sin \theta$  and find a **positive** value of  $\theta$  at which this maximum value occurs. (3 marks)

4 On 1 January 1900, a sculpture was valued at £80.

When the sculpture was sold on 1 January 1956, its value was £5000.

The value, £ $V$ , of the sculpture is modelled by the formula  $V = Ak^t$ , where  $t$  is the time in years since 1 January 1900 and  $A$  and  $k$  are constants.

- (a) Write down the value of  $A$ . (1 mark)
- (b) Show that  $k \approx 1.07664$ . (3 marks)
- (c) Use this model to:
- (i) show that the value of the sculpture on 1 January 2006 will be greater than £200 000; (2 marks)
- (ii) find the year in which the value of the sculpture will first exceed £800 000. (3 marks)

5 (a) (i) Obtain the binomial expansion of  $(1 - x)^{-1}$  up to and including the term in  $x^2$ . (2 marks)

(ii) Hence, or otherwise, show that

$$\frac{1}{3 - 2x} \approx \frac{1}{3} + \frac{2}{9}x + \frac{4}{27}x^2$$

for small values of  $x$ . (3 marks)

(b) Obtain the binomial expansion of  $\frac{1}{(1 - x)^2}$  up to and including the term in  $x^2$ . (2 marks)

(c) Given that  $\frac{2x^2 - 3}{(3 - 2x)(1 - x)^2}$  can be written in the form  $\frac{A}{(3 - 2x)} + \frac{B}{(1 - x)} + \frac{C}{(1 - x)^2}$ ,  
find the values of  $A$ ,  $B$  and  $C$ . (5 marks)

(d) Hence find the binomial expansion of  $\frac{2x^2 - 3}{(3 - 2x)(1 - x)^2}$  up to and including the term in  $x^2$ . (3 marks)

**Turn over for the next question**

**Turn over ►**

6 (a) Express  $\cos 2x$  in the form  $a \cos^2 x + b$ , where  $a$  and  $b$  are constants. (2 marks)

(b) Hence show that  $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx = \frac{\pi}{a}$ , where  $a$  is an integer. (5 marks)

7 The quadrilateral  $ABCD$  has vertices  $A(2, 1, 3)$ ,  $B(6, 5, 3)$ ,  $C(6, 1, -1)$  and  $D(2, -3, -1)$ .

The line  $l_1$  has vector equation  $\mathbf{r} = \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ .

(a) (i) Find the vector  $\overrightarrow{AB}$ . (2 marks)

(ii) Show that the line  $AB$  is parallel to  $l_1$ . (1 mark)

(iii) Verify that  $D$  lies on  $l_1$ . (2 marks)

(b) The line  $l_2$  passes through  $D(2, -3, -1)$  and  $M(4, 1, 1)$ .

(i) Find the vector equation of  $l_2$ . (2 marks)

(ii) Find the angle between  $l_2$  and  $AC$ . (3 marks)

8 (a) Solve the differential equation

$$\frac{dx}{dt} = -2(x - 6)^{\frac{1}{2}}$$

to find  $t$  in terms of  $x$ , given that  $x = 70$  when  $t = 0$ . (6 marks)

(b) Liquid fuel is stored in a tank. At time  $t$  minutes, the depth of fuel in the tank is  $x$  cm. Initially there is a depth of 70 cm of fuel in the tank. There is a tap 6 cm above the bottom of the tank. The flow of fuel out of the tank is modelled by the differential equation

$$\frac{dx}{dt} = -2(x - 6)^{\frac{1}{2}}$$

(i) Explain what happens when  $x = 6$ . (1 mark)

(ii) Find how long it will take for the depth of fuel to fall from 70 cm to 22 cm. (2 marks)

**END OF QUESTIONS**