



**General Certificate of Education (A-level)  
2011**

**Mathematics**

**MS2B**

**(Specification 6360)**

**Statistics 2B**

***Mark Scheme***

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Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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**Key to mark scheme abbreviations**

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct $x$ marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**



**MS2B(cont)**

Question	Solution	Marks	Total	Comments																														
2(a)	<table border="1"> <thead> <tr> <th></th> <th>C</th> <th>L</th> <th>LD</th> <th>OP</th> <th>Tot</th> </tr> </thead> <tbody> <tr> <th>M</th> <td>156</td> <td>144</td> <td>120</td> <td>60</td> <td>480</td> </tr> <tr> <th>F</th> <td>216</td> <td>135</td> <td>108</td> <td>81</td> <td>540</td> </tr> <tr> <th>Tot</th> <td>372</td> <td>279</td> <td>228</td> <td>141</td> <td>1020</td> </tr> </tbody> </table>		C	L	LD	OP	Tot	M	156	144	120	60	480	F	216	135	108	81	540	Tot	372	279	228	141	1020	B1	2	For each correct row						
		C	L	LD	OP	Tot																												
M	156	144	120	60	480																													
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Tot	372	279	228	141	1020																													
		B1																																
(b)	<p><math>H_0</math>: No association between gender and the way students vote</p> <p><math>H_1</math>: Association between gender and the way students vote</p>	B1		For at least $H_0$ correct																														
	<table border="1"> <thead> <tr> <th><math>O_i</math></th> <th><math>E_i</math></th> <th><math>(O_i - E_i)^2 / E_i</math></th> </tr> </thead> <tbody> <tr> <td>156</td> <td>175.06</td> <td>2.075</td> </tr> <tr> <td>216</td> <td>196.94</td> <td>1.844</td> </tr> <tr> <td>144</td> <td>131.29</td> <td>1.230</td> </tr> <tr> <td>135</td> <td>147.71</td> <td>1.093</td> </tr> <tr> <td>120</td> <td>107.29</td> <td>1.505</td> </tr> <tr> <td>108</td> <td>120.71</td> <td>1.337</td> </tr> <tr> <td>60</td> <td>66.35</td> <td>0.608</td> </tr> <tr> <td>81</td> <td>74.65</td> <td>0.541</td> </tr> <tr> <td></td> <td></td> <td><math>X^2 = 10.233</math></td> </tr> </tbody> </table>	$O_i$	$E_i$	$(O_i - E_i)^2 / E_i$	156	175.06	2.075	216	196.94	1.844	144	131.29	1.230	135	147.71	1.093	120	107.29	1.505	108	120.71	1.337	60	66.35	0.608	81	74.65	0.541			$X^2 = 10.233$	M1		Attempt at $E_i$
	$O_i$	$E_i$	$(O_i - E_i)^2 / E_i$																															
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		$X^2 = 10.233$																																
		M1		Attempt at $(O_i - E_i)^2 / E_i$																														
		M1		Attempt at $\sum \left( \frac{(O_i - E_i)^2}{E_i} \right)$																														
		A1		awfw 10.2 to 10.3 (A1 dependent on all 3 method marks)																														
	<p><math>\nu = 3 \Rightarrow \chi_{crit}^2 = 11.345</math></p>	B1		ft on their $\nu$ value																														
		B1ft																																
	<p><math>X^2 &lt; \chi_{crit}^2 \therefore</math> accept <math>H_0</math></p>	A1																																
	<p>Accept claim at 1% level. Evidence to suggest that the way students vote is independent of gender.</p>	E1	9																															
	<b>Total</b>		<b>11</b>																															

**MS2B(cont)**

Question	Solution	Marks	Total	Comments
<b>3(a)(i)</b>	$X \sim P_0(0.6)$ $P(X \leq 1) = 0.8781$	B1	1	Awrt 0.878
<b>(ii)</b>	For matches : The number of run outs: $Y \sim P_0(0.15)$ $P(Y \geq 1) = 1 - P(Y = 0)$ $= 1 - e^{-0.15}$ } $= 1 - 0.8607$ } $= 0.1393$	M1  A1		must use $P_0(0.15)$ awrt 0.139
	$P(X \leq 1 \text{ and } Y \geq 1) = 0.8781 \times 0.1393$ $= 0.122$	M1 A1	4	their (a)(i) $\times$ their $P(Y \geq 1)$ awrt
<b>(b)</b>	$X$ and $Y$ are independent.  Number of catches and runouts independent	B1	1	
<b>(c)(i)</b>	For Season: $S \sim P_0(9.6)$ $P(S = 10) = \frac{e^{-9.6} \times 9.6^{10}}{10!}$ $= 0.124$	M1  A1	2	Use of $\lambda = 9.6$ in correct Poisson expression
<b>(ii)</b>	$T \sim P_0(9.6 + 2.4) = P_0(12)$ $P(T \geq 15) = 1 - P(T \leq 14)$ $= 1 - 0.7720$ $= 0.228$	B1  B2,1	3	$P_0(12)$ used or seen  ( $1 - 0.8444 = 0.155$ to $0.156$ ) <b>B1</b>
	<b>Total</b>		<b>11</b>	

**MS2B(cont)**

Question	Solution	Marks	Total	Comments																															
<b>4(a)(i)</b>	$E(X) = \sum_x x \times P(X = x) = 2.8$	B1																																	
	$E(X^2) = \sum_x x^2 \times P(X = x) = 9$																																		
	$Var(X) = 9 - 2.8^2$ $= 1.16$	M1 A1	<b>3</b>	(their $E(X^2)$ – their $E^2(X)$ ) cao																															
<b>(ii)</b>	$E(S) = 3 \times E(X) = 8.4$	B1ft		on their $E(X)$																															
	$Var(S) = 3 \times Var(X) = 3.48$	B1ft	<b>2</b>	$3 \times$ their $Var(X)$ from (i) $> 0$ <b>NB</b> There was a problem with part 4(a)(ii) which affected the marking of this part. Please see the Report on the Examination for details.																															
<b>(b)</b>	$E(Y) = 3.5$																																		
	$E(Y^2) = 13$	B1		for $E(Y)$																															
	$Var(Y) = 13 - 3.5^2 = 0.75$	M1 A1ft		on their $E(Y)$ and $E(Y^2)$ $Var(Y) > 0$																															
	$E(T) = 3 \times E(Y) = 10.5$	B1		cao																															
	$Var(T) = 3^2 \times Var(Y)$ $= 9 \times 0.75$ $= 6.75$	M1 A1		$9 \times$ their $Var(Y) > 0$ cao																															
	<b>Alternative:</b>																																		
	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td><i>T</i></td> <td>3</td> <td>6</td> <td>9</td> <td>12</td> <td></td> </tr> <tr> <td><i>T</i><sup>2</sup></td> <td>9</td> <td>36</td> <td>81</td> <td>144</td> <td></td> </tr> <tr> <td><i>P</i></td> <td><math>\frac{1}{20}</math></td> <td><math>\frac{2}{20}</math></td> <td><math>\frac{3}{20}</math></td> <td><math>\frac{4}{20}</math></td> <td></td> </tr> <tr> <td><math>E(T)</math></td> <td><math>\frac{3}{20}</math></td> <td><math>\frac{12}{20}</math></td> <td><math>\frac{27}{20}</math></td> <td><math>\frac{168}{20}</math></td> <td>10.5</td> </tr> <tr> <td><math>E(T^2)</math></td> <td><math>\frac{9}{20}</math></td> <td><math>\frac{72}{20}</math></td> <td><math>\frac{243}{20}</math></td> <td><math>\frac{2016}{20}</math></td> <td>117</td> </tr> </table>	<i>T</i>	3	6	9	12		<i>T</i> <sup>2</sup>	9	36	81	144		<i>P</i>	$\frac{1}{20}$	$\frac{2}{20}$	$\frac{3}{20}$	$\frac{4}{20}$		$E(T)$	$\frac{3}{20}$	$\frac{12}{20}$	$\frac{27}{20}$	$\frac{168}{20}$	10.5	$E(T^2)$	$\frac{9}{20}$	$\frac{72}{20}$	$\frac{243}{20}$	$\frac{2016}{20}$	117	(M1A1)			
	<i>T</i>	3	6	9	12																														
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$E(T^2)$	$\frac{9}{20}$	$\frac{72}{20}$	$\frac{243}{20}$	$\frac{2016}{20}$	117																														
		(M1A1)																																	
	$Var(T) = E(T^2) - [E(T)]^2$ $= 117 - 10.5^2$ $= 6.75$	(M1) (A1)	<b>6</b>	(used)																															

**MS2B(cont)**

Question	Solution	Marks	Total	Comments
<b>4(c)(i)</b>	$P(X > 1) = 0.8$	B1	1	
<b>(ii)</b>	$P(T = 3) = \frac{1}{20} \text{ and } P(T = 3 \text{ or } 6) = \frac{3}{20}$ $P(X + T \leq 9 \text{ and } X > 1)$ $= P([2, 3 \text{ or } 6], [3, 3 \text{ or } 6], [4, 3])$ $= 0.1 \times \frac{3}{20} + 0.4 \times \frac{3}{20} + 0.3 \times \frac{1}{20}$ $= 0.015 + 0.06 + 0.015$ $= 0.09$	B4	4	<p><b>Alternative:</b>  <math>P(T = 3) = \frac{1}{20}</math> and <math>P(T = 6) = \frac{2}{20}</math>  <math>P(X + T \leq 9 \text{ and } X &gt; 1)</math>  <math>= P([(2, 3, 4), 3], [(2, 3), 6])</math>  <math>= 0.8 \times \frac{1}{20} + 0.5 \times \frac{2}{20}</math>  <math>= 0.04 + 0.05 = 0.09</math></p> <p><b>sc</b>                      [any 4 correct p's from table B2]                      [0.12 <b>B3</b>] [0.096 <b>or</b> 0.072 <b>B2</b>]</p> $\left\{ \begin{array}{l} P(T = 3) = \frac{1}{20} \text{ and } P(T = 3 \text{ or } 6) = \frac{3}{20} \\ P(T = 3) = \frac{1}{20} \text{ and } P(T = 6) = \frac{2}{20} \end{array} \right.$ <p style="text-align: right;"><b>B1</b></p>
<b>(iii)</b>	$P(X + T \leq 9   X > 1) = \frac{0.09}{0.80}$ $= \frac{9}{80} \text{ (0.1125)}$	M1 A1	2	<p>their (c)(ii) <math>\frac{\quad}{0.80}</math> (<math>0 &lt; p &lt; 1</math>)</p> <p>cao</p>
<b>Total</b>			<b>18</b>	



## MS2B(cont)

Question	Solution	Marks	Total	Comments						
5(a)(i)	$H_0: \mu = 165$ $H_1: \mu > 165$	B1	1							
(ii)	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%;">David (5%)</td> <td style="width: 50%;">James (1%)</td> </tr> <tr> <td colspan="2" style="text-align: center;"> <math display="block">z = \frac{167.1 - 165}{\sqrt{101.2} / 10}</math> <math display="block">= 2.09</math> </td> </tr> <tr> <td> <math>z_{crit} = 1.6449</math>  <math>(t_{crit} = 1.660)</math>                      Reject <math>H_0</math>                       Evidence to suggest that the <b>mean</b> height of students in final year has <b>increased</b> at 5% level                 </td> <td> <math>z_{crit} = 2.3263</math>  <math>(t_{crit} = 2.364)</math>                      Accept <math>H_0</math>                       No evidence to suggest an <b>increase</b> in the <b>mean</b> height of final year students at 1% level                 </td> </tr> </table>	David (5%)	James (1%)	$z = \frac{167.1 - 165}{\sqrt{101.2} / 10}$ $= 2.09$		$z_{crit} = 1.6449$ $(t_{crit} = 1.660)$ Reject $H_0$  Evidence to suggest that the <b>mean</b> height of students in final year has <b>increased</b> at 5% level	$z_{crit} = 2.3263$ $(t_{crit} = 2.364)$ Accept $H_0$  No evidence to suggest an <b>increase</b> in the <b>mean</b> height of final year students at 1% level	B1  M1 A1  B1 A1  E1 E1	6	awfw 2.08 to 2.09  (both) (both) dependent on M1
David (5%)	James (1%)									
$z = \frac{167.1 - 165}{\sqrt{101.2} / 10}$ $= 2.09$										
$z_{crit} = 1.6449$ $(t_{crit} = 1.660)$ Reject $H_0$  Evidence to suggest that the <b>mean</b> height of students in final year has <b>increased</b> at 5% level	$z_{crit} = 2.3263$ $(t_{crit} = 2.364)$ Accept $H_0$  No evidence to suggest an <b>increase</b> in the <b>mean</b> height of final year students at 1% level									
(iii)	<p><b>Population not stated as being Normal / not known.</b></p> <p>Heights of all students may not be Normal/ Known</p>	B1	1	Large sample size of 100 indicates that the distribution of the sample mean is very likely to be Normal even though <b>the parent population not given as being Normal.</b> Hence $\bar{X} \sim N\left(\mu, \frac{s^2}{n}\right)$						
(b)(i)	David: $\mu = 165$ $\therefore$ rejected $H_0$ when $H_0$ correct $\Rightarrow$ Type I error	M1 A1								
(ii)	James: $\mu = 165$ $\therefore$ accepted $H_0$ when $H_0$ correct $\Rightarrow$ No error	M1 A1	4							
<b>Total</b>			<b>12</b>							

## MS2B(cont)

Question	Solution	Marks	Total	Comments
6(a)		B3	3	B1 for concave curve from $(0, 1)$ to $\left(\frac{1}{2}, \frac{3}{32}\right)$ B1 for horizontal straight line $f = \frac{3}{32}$ from $\left(\frac{1}{2}, \frac{3}{32}\right)$ to $\left(\frac{1}{2}, \frac{3}{32}\right)$ B1 for correct axes
(b)(i)	$P\left(X \geq 8\frac{1}{3}\right) = \left[\frac{3}{32} \times \left(11 - 8\frac{1}{3}\right)\right]$ $= \frac{3}{32} \times \frac{8}{3}$ $= \frac{1}{4}$	M1  A1		Any correct method attempted in either part  AG
(ii)	$P(X \geq 3) = \frac{3}{32} \times (11 - 3)$ $= \frac{3}{4}$	A1	3	Any correct method attempted  AG
(c)(i)	Interquartile Range = $5\frac{1}{3}$	B1		cao
(ii)	Median = $5\frac{2}{3}$  <b>Alternative :</b> $\frac{1}{64} + \frac{3}{32} \left(m - \frac{1}{2}\right) = \frac{1}{2}$ $\Rightarrow 3 \left(m - \frac{1}{2}\right) = 15.5 \Rightarrow m = 5\frac{2}{3}$	B2	3	cao  sc if B0 then: M1 for correct method seen $\frac{1}{2} \left(8\frac{1}{3} + 3\right)$ or $\frac{1}{2} \times 11\frac{1}{3}$ or $\frac{3}{32} (11 - m) = \frac{1}{2} \Rightarrow 11 - 5\frac{1}{3}$
(d)	$P[(X < m) \cap (X \geq 3)] = \frac{1}{4}$ $P(X < m   X \geq 3) = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$	B1  M1  A1	3	$\left(\frac{3}{4} - \frac{1}{2}\right)$ attempted  (their $p$ ) $\frac{1}{3/4}$ for $0 < p < 1$ cao  <b>Alternative:</b> (Ratio of relevant two areas) $P(X < m   X \geq 3) = \frac{2\frac{2}{3}}{8} = \frac{1}{3}$ cao
	<b>Total</b>		<b>12</b>	
	<b>TOTAL</b>		<b>75</b>	