
A-level
MATHEMATICS
7357/2

Paper 2

Mark scheme

June 2021

Version 1.1 Final Mark Scheme



Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

| | |
|---|---|
| M | mark is for method |
| R | mark is for reasoning |
| A | mark is dependent on M marks and is for accuracy |
| B | mark is independent of M marks and is for method and accuracy |
| E | mark is for explanation |
| F | follow through from previous incorrect result |

Key to mark scheme abbreviations

| | |
|---------|---|
| CAO | correct answer only |
| CSO | correct solution only |
| ft | follow through from previous incorrect result |
| ‘their’ | Indicates that credit can be given from previous incorrect result |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| NMS | no method shown |
| PI | possibly implied |
| sf | significant figure(s) |
| dp | decimal place(s) |

AS/A-level Maths/Further Maths assessment objectives

| AO | | Description |
|------------|--------|---|
| AO1 | AO1.1a | Select routine procedures |
| | AO1.1b | Correctly carry out routine procedures |
| | AO1.2 | Accurately recall facts, terminology and definitions |
| AO2 | AO2.1 | Construct rigorous mathematical arguments (including proofs) |
| | AO2.2a | Make deductions |
| | AO2.2b | Make inferences |
| | AO2.3 | Assess the validity of mathematical arguments |
| | AO2.4 | Explain their reasoning |
| | AO2.5 | Use mathematical language and notation correctly |
| AO3 | AO3.1a | Translate problems in mathematical contexts into mathematical processes |
| | AO3.1b | Translate problems in non-mathematical contexts into mathematical processes |
| | AO3.2a | Interpret solutions to problems in their original context |
| | AO3.2b | Where appropriate, evaluate the accuracy and limitations of solutions to problems |
| | AO3.3 | Translate situations in context into mathematical models |
| | AO3.4 | Use mathematical models |
| | AO3.5a | Evaluate the outcomes of modelling in context |
| | AO3.5b | Recognise the limitations of models |
| | AO3.5c | Where appropriate, explain how to refine models |

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

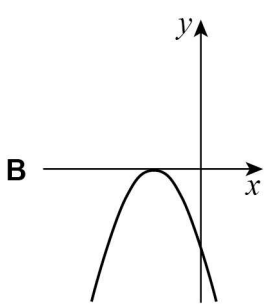
Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

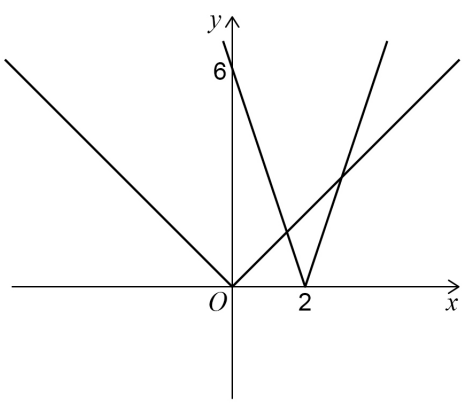
Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

| Q | Marking instructions | AO | Marks | Typical solution |
|-------|----------------------|------|-------|---|
| 1 | Ticks correct box | 2.2a | B1 |  |
| Total | | | 1 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|-------|------------------------|-----|-------|------------------|
| 2 | Circles correct answer | 1.2 | B1 | $f''(7) = 0$ |
| Total | | | 1 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|-------|------------------------|------|-------|------------------|
| 3 | Circles correct answer | 2.2a | B1 | a |
| Total | | | 1 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|------|--|------|----------|---|
| 4(a) | Sketches any V shaped graph with the apex on the positive x axis | 1.1a | M1 |  |
| | Sketches a roughly symmetrical v-shaped graph touching the positive x -axis and intersecting $y = 2x $ twice in the first quadrant Condone missing or incorrect labels on the axes | 1.1b | A1 | |
| | Subtotal | | 2 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|------|---|------|----------|---|
| 4(b) | Forms the equation $ 3x - 6 = 2x $ and selects an appropriate method to begin removing modulus signs For example Squares both sides to obtain $9x^2 - 36x + 36 = 4x^2$ or Considers $3x - 6 = 2x$ or $3x - 6 = -2x$ | 3.1a | M1 | $ 3x - 6 = 2x $ $3x - 6 = 2x$ $x = 6$ $-3x + 6 = 2x$ $x = 1.2$ when $x = 6$ $y = 12$ when $x = 1.2$ $y = 2.4$ |
| | Obtains $x = 6$ | 1.1b | A1 | |
| | Obtains $x = 1.2$ OE | 1.1b | A1 | |
| | Obtains $y = 12$ and $y = 2.4$ | 1.1b | A1 | |
| | Subtotal | | 4 | |

| | | | | |
|--|-----------------------|--|----------|--|
| | Question Total | | 6 | |
|--|-----------------------|--|----------|--|

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|------|----------|---|
| 5 | Forms the identity/equation $5(x - 3) \equiv A(4 - 3x) + B(2x - 11)$ and either Compares coefficients or Substitutes a value for x PI by correct A or B | 1.1a | M1 | $\frac{5(x - 3)}{(2x - 11)(4 - 3x)} = \frac{A}{(2x - 11)} + \frac{B}{(4 - 3x)}$ $5(x - 3) = A(4 - 3x) + B(2x - 11)$ $x = \frac{4}{3} \Rightarrow B = 1$ $x = \frac{11}{2} \Rightarrow A = -1$ |
| | Obtains $A = -1$ | 1.1b | A1 | |
| | Obtains $B = 1$ | 1.1b | A1 | |
| | Total | | 3 | |

| Q | Marking instructions | AO | Mark | Typical solution |
|---|--|------|----------|---|
| 6 | Takes logs of both sides and uses a log rule correctly | 1.1a | M1 | $\ln 5^x = \ln 3^{x+4}$ |
| | Applies all necessary log rules correctly so that x is no longer an exponent and expresses $4 \ln 3$ in terms of x Condone sign error | 1.1a | M1 | $x \ln 5 = (x + 4) \ln 3$ $x \ln 5 - x \ln 3 = 4 \ln 3$ $x(\ln 5 - \ln 3) = \ln 81$ |
| | Obtains $\ln 81$ from $4 \ln 3$ or from $3^x \times 3^4$ | 1.1b | B1 | $x = \frac{\ln 81}{\ln 5 - \ln 3}$ |
| | Completes reasoned argument to show given result Must see $x(\ln 5 - \ln 3)$ on the penultimate line If natural logs are not used throughout then the base must be converted at the end to get this mark | 2.1 | R1 | |
| | Total | | 4 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|-----------------|--|------|----------|------------------|
| 7(a)(i) | Obtains centre = (3,4) Accept $a = 3$, $b = 4$ | 1.1b | B1 | Centre (3,4) |
| Subtotal | | | 1 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|-----------------|--|------|----------|---|
| 7(a)(ii) | Rearranges into a standard form with $(x \pm 3)^2 + (y \pm 4)^2$ seen or Forms an expression for radius of the form $\sqrt{(\pm 3)^2 + (\pm 4)^2 \pm p}$ This working can be seen in part (a)(i) | 1.1a | M1 | $x^2 + y^2 = 6x + 8y + p$ $x^2 + y^2 - 6x - 8y - p = 0$ $(x - 3)^2 + (y - 4)^2 - 9 - 16 - p = 0$ $(x - 3)^2 + (y - 4)^2 = 25 + p$ |
| | Obtains $(x - 3)^2 + (y - 4)^2 = 25 + p$ Or Obtains $\sqrt{3^2 + 4^2 + p}$ | 1.1b | A1 | Radius = $\sqrt{25 + p}$ |
| | States radius = $\sqrt{25 + p}$ | 1.1b | A1 | |
| Subtotal | | | 3 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|------|---|------|-------|--|
| 7(b) | Begins to solve the problem by either Sketching (part of) a circle which goes through the origin or Sketching (part of) a circle that touches one of the axes or Substituting either $x = 0$ or $y = 0$ into their circle equation – must involve p PI by correctly formed equation involving p | 3.1a | M1 | Circle passes through the origin $\sqrt{25 + p} = 5$ $\Rightarrow p = 0$ Circle just touches x -axis $\sqrt{25 + p} = 4$ $\Rightarrow p = -9$ |
| | Forms an equation to find p By either Equating their expression for the radius to 5 or the greater value of their a and b or Substituting both $x = 0$ and $y = 0$ into their circle equation to form an equation in terms of p only or Substituting $y = 0$ and using $b^2 - 4ac = 0$ to form an equation in terms of p only | 3.1a | M1 | |

| | | | | |
|--|------------------|------|----------|--|
| | Deduces $p = 0$ | 2.2a | R1 | |
| | Deduces $p = -9$ | 2.2a | R1 | |
| | Subtotal | | 4 | |

| | | | | |
|--|-----------------------|--|----------|--|
| | Question Total | | 8 | |
|--|-----------------------|--|----------|--|

| Q | Marking instructions | AO | Marks | Typical solution |
|-------------|---|-----|----------|---|
| 8(a) | Identifies the algebraic mistake | 2.3 | E1 | Kai has not expanded the brackets correctly |
| | Identifies the mistake that all cases have not been exhausted | 2.3 | E1 | Kai has not considered numbers of the form $3m+2$ |
| | Subtotal | | 2 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|-------------|--|------|----------|---|
| 8(b) | Completes manipulation correctly in Step 4 to obtain $(3m + 1)(9m^2 + 6m)$ OE | 2.1 | B1 | Step 4: $= (3m + 1)(9m^2 + 6m + 1 - 1)$ $= (3m + 1)(9m^2 + 6m)$ $= 3(3m + 1)(3m^2 + 2m)$ which is a multiple of 3 |
| | Manipulates expression with a third substitution using either $n = 3m + 2$ or $n = 3m - 1$ | 1.1a | M1 | Step 5: when $n = 3m + 2$, $n^3 - n = (3m + 2)((3m + 2)^2 - 1)$ $= (3m + 2)(9m^2 + 12m + 3)$ $= 3(3m + 2)(3m^2 + 4m + 1)$ which is a multiple of 3 |
| | Manipulates correct expression to convincingly show multiple of 3 Equivalent working for $n = 3m - 1$ $n^3 - n$ $= (3m - 1)((3m - 1)^2 - 1)$ $= (3m - 1)(9m^2 - 6m)$ $= 3(3m - 1)(3m^2 - 2m)$ OE | 1.1b | A1 | |
| | Completes rigorous argument by clearly showing the factor of 3 in the two cases required and concludes appropriately | 2.1 | R1 | $n^3 - n$ is always a multiple of 3 for all positive integer values of n |
| | Subtotal | | 4 | |

| | | | | |
|--|-----------------------|--|----------|--|
| | Question Total | | 6 | |
|--|-----------------------|--|----------|--|

| Q | Marking instructions | AO | Marks | Typical solution |
|----------|---|-----|-------|---|
| 9(a) | Begins to find the required horizontal distance by considering an appropriate horizontal distance in a right-angled triangle For example: $d \cos \theta$, $d \cos 2\theta$ or $d \sin\left(\frac{\pi}{2} - 2\theta\right)$ Award for correctly identifying at least one of the horizontal components required. This can be done on a sketch or by clearly stating which distance they are referring to | 3.3 | M1 | QP makes angle of 2θ with horizontal $x = d \cos \theta - d \cos 2\theta$ $x = d \cos \theta - d \sin\left(\frac{\pi}{2} - 2\theta\right)$ $x = d\left(\cos \theta + \sin\left(2\theta - \frac{\pi}{2}\right)\right)$ |
| | Completes correct manipulation to show required result AG | 2.1 | R1 | |
| Subtotal | | | 2 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|----------|---|------|-------|---|
| 9(b) | Uses a compound angle formula and expands $\sin\left(2\theta - \frac{\pi}{2}\right)$ or Uses complementary angles to obtain the formula without $\frac{\pi}{2}$ or States $x = d(\cos \theta - \cos 2\theta)$ | 3.1a | M1 | $x = d\left(\cos \theta + \sin\left(2\theta - \frac{\pi}{2}\right)\right)$ $x = d(\cos \theta - \cos 2\theta)$ $x = d(\cos \theta - (2\cos^2 \theta - 1))$ $x = d(1 + \cos \theta - 2\cos^2 \theta)$ |
| | Uses $\cos 2\theta = 2\cos^2 \theta - 1$ to show the required result AG | 2.1 | R1 | |
| Subtotal | | | 2 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|----------|--|------|-------|--|
| 9(c) | States greatest value = $\frac{9d}{8}$ | 1.1b | B1 | Greatest value = $\frac{9d}{8}$ $\cos \theta = \frac{1}{4}$ |
| | States $\cos \theta = \frac{1}{4}$ | 1.1b | B1 | |
| Subtotal | | | 2 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|------|---|------|----------|--|
| 9(d) | Begins to find OQ by either using the cosine rule or the sine rule with d and θ Accept either $OQ^2 = d^2 + d^2 - 2d^2 \cos \theta$ or $\frac{OQ}{\sin \theta} = \frac{d}{\sin \alpha}$ Note $\alpha = \frac{\pi - \theta}{2}$ | 3.1a | M1 | $OQ^2 = d^2 + d^2 - 2d^2 \cos \theta$ $= 2d^2 - 2d^2 \times \frac{1}{4}$ $= \frac{3d^2}{2}$ $OQ = \frac{\sqrt{6}}{2}d$ |
| | Substitutes their exact value for $\cos \theta$ into the cosine rule or Finds and substitutes their corresponding exact values for $\sin \theta$ and $\cos \frac{\theta}{2}$ into the sine rule Note When $\cos \theta = \frac{1}{4}$, $\sin \theta = \frac{\sqrt{15}}{4}$ and $\cos \frac{\theta}{2} = \sqrt{\frac{5}{8}}$ | 1.1a | M1 | |
| | Obtains the correct exact value of OQ ACF | 1.1b | A1 | |
| | Subtotal | | 3 | |
| | Question Total | | 9 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|----------|--|------|-------|-----------------------------|
| 10(a) | Obtains a domain excluding negatives or excluding 3 Condone $x > 0$ | 1.1a | M1 | $\{x: x \geq 0, x \neq 3\}$ |
| | Deduces both $x \geq 0$ and $x \neq 3$ with no extras Condone $x > 0$ | 2.2a | A1 | |
| | Obtains correct domain correctly stated in set notation | 2.5 | R1 | |
| Subtotal | | | 3 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|----------|--|-----|-------|--|
| 10(b) | States that $h(x)$ has a discontinuity/asymptote at $x = 3$ or in the interval (1, 4) OE | 2.4 | M1 | $h(x)$ is not continuous at $x = 3$ |
| | Explains that the discontinuity is at $x = 3$ and this is in the interval (1, 4) | 2.3 | A1 | This means that a change of sign between $x = 1$ and 4 does not imply a root |
| Subtotal | | | 2 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|----------|---|------|-------|---|
| 10(c) | Selects an appropriate method to differentiate and reaches $h'(x)$ of the form $ax^{-\frac{1}{2}}(x-3)^{-1} + bx^{\frac{1}{2}}(x-3)^{-2}$ OE | 3.1a | M1 | $h(x) = \frac{\sqrt{x}}{x-3}$ $h'(x) = \frac{\frac{1}{2}x^{-\frac{1}{2}}(x-3) - x^{\frac{1}{2}}}{(x-3)^2}$ $h'(x) = 0 \Rightarrow \frac{\frac{1}{2}x^{-\frac{1}{2}}(x-3) - x^{\frac{1}{2}}}{(x-3)^2} = 0$ $\frac{1}{2}x^{-\frac{1}{2}}(x-3) - x^{\frac{1}{2}} = 0$ $\frac{(x-3)}{2\sqrt{x}} - \sqrt{x} = 0$ $x-3-2x = 0$ $x = -3$ <p>$x = -3$ is not in the domain of h hence the function has no turning points</p> <p>$h(x) > 0$ for $x > 3$ $h(x) < 0$ for $x < 3$ Hence function is one to one and has an inverse</p> |
| | Obtains correct $h'(x)$ ACF | 1.1b | A1 | |
| | Equates their $h'(x)$ to 0 | 1.1a | M1 | |
| | Obtains $x = -3$ | 1.1b | A1 | |
| | Explains that a continuous function with no turning points is one to one and therefore the inverse exists. Condone omission of 'continuous' | 2.4 | E1 | |
| | Completes a reasoned argument to correctly show that the function is one to one and deduces that $h(x)$ has an inverse Must explain that $x = -3$ is not in the domain and therefore there are no turning points and considers the sign of $h(x)$ either side of $x = 3$ | 2.1 | R1 | |
| Subtotal | | | 6 | |

| | | | | |
|--|-----------------------|--|-----------|--|
| | Question Total | | 11 | |
|--|-----------------------|--|-----------|--|

| Q | Marking Instructions | AO | Marks | Typical Solution |
|-----------|-----------------------------|-----------|--------------|-------------------------|
| 11 | Circles correct answer | 1.1b | B1 | $v = 1.5e^{0.5t}$ |
| | Total | | 1 | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|-----------|-----------------------------|-----------|--------------|-------------------------|
| 12 | Circles correct answer | 1.1b | B1 | $v_2 = 6 \sin 30^\circ$ |
| | Total | | 1 | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|-----------|--|-----------|--------------|--|
| 13 | Selects and uses an appropriate constant acceleration equation to find the acceleration. Must have substituted at least two values correctly | 3.1b | M1 | $v^2 = u^2 + 2as$ $289 = 169 + 80a$ $\Rightarrow a = 1.5$ $F = 1200 \times 1.5 = 1800 \text{ N}$ |
| | Obtains $a = 1.5$ PI by correct F | 1.1b | A1 | |
| | Substitutes their value of a into $F = ma$ to obtain their F Condone missing or incorrect units | 1.1b | B1F | |
| | Total | | 3 | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|-----------|--|-----------|--------------|--|
| 14 | Interprets the problem to consider the area beneath the curve | 3.1b | M1 | Distance = area below the curve Split into four trapezia |
| | Divides an appropriate area into at least three polygons and finds at least one correct area for their chosen method | 1.1b | A1 | Trapezium 1 = $\frac{6}{2}(5.8 + 5.2) = 33$ Trapezium 2 = $\frac{7}{2}(5.2 + 6.2) = 39.9$ |
| | Obtains a total area for the interval $12 < t < 36$ using their method AWFW 125 –135 | 1.1a | M1 | Trapezium 3 = $\frac{5}{2}(6.2 + 6) = 30.5$ |
| | Compares their total area with Noosha's estimate to conclude that her result was reasonable | 1.1b | R1 | Trapezium 4 = $\frac{6}{2}(6 + 3.8) = 30.5$ Total = $132.8 \approx 130$ Noosha's estimate was reasonable |
| | Total | | 4 | |

| Q | Marking Instructions | AO | Marks | Typical Solution |
|-------|--|------|----------|---|
| 15(a) | Deduces $T = 12$ | 2.2a | B1 | $T = 12$ |
| | States clear reason For example: No resultant force since no acceleration T must balance the resistant force as speed is constant | 2.4 | E1 | Constant speed means forces on trailer are in equilibrium |
| | Subtotal | | 2 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|-------|--|------|----------|----------------------------|
| 15(b) | States one valid assumption For example: Rod is rigid Rod lies parallel to the direction of travel Rod is inextensible | 3.5b | E1 | The rod remains horizontal |
| | Subtotal | | 1 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|-------|--|------|----------|------------------------------------|
| 15(c) | Forms equilibrium equation of forces acting on cyclist and cycle PI by $R = 28$ | 3.3 | M1 | $40 = T + R$ $R = 28 \text{ N}$ |
| | Obtains resistance force = 28 N Must state units | 3.2a | A1 | |
| | Subtotal | | 2 | |

| | | | | |
|--|-----------------------|--|----------|--|
| | Question Total | | 5 | |
|--|-----------------------|--|----------|--|

| Q | Marking Instructions | AO | Marks | Typical Solution |
|----|--|------|----------|---|
| 16 | <p>Takes moments about the point of suspension to form a moments equation involving w and x with one term correctly expressed.</p> <p>or</p> <p>Takes moments about any other point to form a moments equation involving w, x and T with one term correctly expressed.</p> <p>If moments taken about A then $0.2g(3) + 3w(6) = T(6 - x)$</p> <p>If moments taken about B then $w(6) + 0.2g(3) = Tx$</p> | 3.3 | M1 | <p>Take moments about the point of suspension</p> $3wx = (3 - x)0.2g + (6 - x)w$ $4wx + 0.2gx = 6w + 0.6g$ $2(2w + 0.1g)x = 6w + 0.6g$ $\therefore x = \frac{3w + 0.3g}{2w + 0.1g}$ |
| | Forms a dimensionally correct moments equation with two terms correctly expressed | 1.1a | M1 | |
| | Forms a fully correct equation involving w , x and g only If moments taken about A or B then $T = 4w + 0.2g$ must have been substituted at this point | 1.1b | A1 | |
| | <p>Obtains given answer showing at least the step</p> $x(4w + 0.2g) = 6w + 0.6g$ <p>OE</p> <p>or</p> $x = \frac{6w + 0.6g}{4w + 0.2g}$ <p>OE</p> <p>Must have fully corrected any error involving weight and mass throughout</p> <p>AG</p> | 2.1 | R1 | |
| | Total | | 4 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|----------|--|------|-------|--|
| 17(a) | Selects and uses an appropriate constant acceleration equation or Uses integration by stating that $a = g$ and integrating to obtain $v = gt + c$ | 1.1a | M1 | $v = u + at$ $u = 0 \quad t = 2 \text{ and } a = g$ $v = 0 + g \times 2 = 2g \text{ m s}^{-1}$ |
| | Completes reasoned argument by stating $u = 0$ and substituting the correct values for u, a and t to obtain the given answer Must have used consistent signs for v and a or Substitutes $v = 0$ and $t = 0$ into $v = gt + c$ to obtain $c = 0$ and concludes that $v = 2g$ AG | 2.1 | R1 | |
| Subtotal | | | 2 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|----------|--|------|-------|--|
| 17(b) | States or uses $a = \frac{dv}{dt}$ or $v = \int a dt$ | 3.4 | B1 | $a = \frac{dv}{dt}$ $\frac{dv}{dt} = g - 0.1v$ $\int \frac{1}{g - 0.1v} dv = \int dt$ $-10 \ln(g - 0.1v) = t + c$ when $t = 0$ then $v = 0$ $-10 \ln g = c$ $-10 \ln(g - 0.1v) = t - 10 \ln g$ $-10 \ln \frac{g - 0.1v}{g} = t$ $g - 0.1v = ge^{-0.1t}$ $v = 10g(1 - e^{-0.1t})$ |
| | Forms differential equation using $\frac{dv}{dt}$ | 3.1a | M1 | |
| | Separates the variables and integrates one side correctly | 1.1a | M1 | |
| | Obtains integral of the form $k \ln(g - 0.1v) = t + c$ Condone missing constant | 1.1b | A1 | |
| | Substitutes the initial conditions to correctly find a value for their c | 3.4 | M1 | |
| | Finds correct constant of integration | 1.1b | A1 | |
| | Obtains $v = 10g(1 - e^{-0.1t})$ OE | 1.1b | A1 | |
| Subtotal | | | 7 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|-------|--|------|-------|--|
| 17(c) | Explains that as t becomes large Andy's model has an increasing velocity | 3.5a | E1 | Under Andy's model the velocity keeps increasing |

| | | | | |
|--|---|------|----------|--|
| | Explains that as t becomes large Amy's model reaches an upper limit | 3.5a | E1 | Under Amy's model the velocity approaches an upper limit |
| | Subtotal | | 2 | |

| | | | | |
|--|-----------------------|--|-----------|--|
| | Question Total | | 11 | |
|--|-----------------------|--|-----------|--|

| Q | Marking Instructions | AO | Marks | Typical Solution |
|-------|---|------|----------|--|
| 18(a) | States or uses an appropriate component of either the horizontal or vertical velocity | 1.1b | B1 | $0 = t_p u \sin \theta - \frac{1}{2} g t_p^2$ $t_p = \frac{2u \sin \theta}{g}$ $t_Q = \frac{4u \sin 2\theta}{g}$ $t_p u \cos \theta = t_Q \times 2u \cos 2\theta$ $\frac{2u^2 \sin \theta \cos \theta}{g} = \frac{8u^2 \sin 2\theta \cos 2\theta}{g}$ $2 \sin \theta \cos \theta = 8 \sin 2\theta \cos 2\theta$ $\frac{1}{8} = \cos 2\theta$ |
| | Considers vertical motion and uses an appropriate constant acceleration equation for one particle | 3.3 | M1 | |
| | Finds a correct expression for t_p and t_Q PI by correct equation for the ranges | 3.4 | A1 | |
| | States horizontal distance $= ut \cos \theta$ or $2ut \cos 2\theta$ | 3.3 | B1 | |
| | Equates their expressions for the ranges of the two particles and substitutes their expressions for t_p and t_Q | 3.4 | M1 | |
| | Completes reasoned argument by simplifying to get an equation in θ only and then using $\sin 2\theta = 2 \sin \theta \cos \theta$ to reach given answer AG | 2.1 | R1 | |
| | Subtotal | | 6 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|----------|---|------|-------|---|
| 18(b) | Obtains exact value for $\cos \theta$ or Obtains $\theta = 41.4^\circ$ AWRT PI by obtaining $t = 1.2$ | 1.1b | B1 | $\cos \theta = \frac{3}{4}$ $t_P \cos \theta = t_Q \times 2 \cos 2\theta$ $\cos \theta = \frac{3}{4}, \cos 2\theta = \frac{1}{8}, t_P = 0.4$ $0.4 \times \frac{3}{4} = t_Q \times 2 \times \frac{1}{8}$ $t_Q = 1.2 \text{ seconds}$ |
| | Uses $t_P \cos \theta = t_Q \times 2 \cos 2\theta$ OE and substitutes $t_P = 0.4$ or $\theta =$ their 41.4 or $\cos \theta =$ their $\frac{3}{4}$ and $\cos 2\theta = \frac{1}{8}$ or Uses $0 = ut_P \sin \theta - \frac{gt_P^2}{2}$ OE and substitutes $t_P = 0.4$ and $\theta =$ their 41.4 or $\sin \theta =$ their $\frac{\sqrt{7}}{4}$ to obtain u AWFW $u = [2.96, 2.97]$ | 3.1b | M1 | |
| | Completes substitution in $t_P u \cos \theta = t_Q \times 2 u \cos 2\theta$ or uses $t_P \cos \theta = t_Q \times 2 \cos 2\theta$ and finds a value for t_Q or Deduces $t_Q = 1.6 \cos \theta$ or $0.625 t_Q = \cos \theta$ OE or Uses $0 = 2ut_Q \sin 2\theta - \frac{gt_Q^2}{2}$ substituting their values of u and θ to find t_Q Note $\sin 2\theta = \frac{3\sqrt{7}}{8}$ | 3.4 | M1 | |
| | Obtains $t_Q = 1.2$ AWRT 1.2 | 1.1b | A1 | |
| Subtotal | | | 4 | |

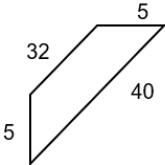
| Q | Marking instructions | AO | Marks | Typical solution |
|----------|--|------|-------|------------------------------------|
| 18(c) | States any suitable assumption For example: Acceleration is constant X and Y are at the same height | 3.5b | E1 | X and Y are at the same height |
| Subtotal | | | 1 | |

| | | | | |
|----------------|--|--|----|--|
| Question Total | | | 11 | |
|----------------|--|--|----|--|

| Q | Marking instructions | AO | Marks | Typical solution |
|----------|---|-----|----------|---|
| 19(a)(i) | Explains that when two vectors are parallel one is a scalar multiple of the other Must refer to scalar multiple or show this algebraically in the form $\mathbf{a} = k\mathbf{b}$ Do not accept 'factor' | 2.4 | E1 | When two vectors are parallel one is a scalar multiple of the other |
| | Subtotal | | 1 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|-----------|-------------------------|------|----------|---|
| 19(a)(ii) | Verifies that $k = 0.8$ | 1.1b | B1 | Amba's velocity = $0.8(2.8\mathbf{i} + 9.6\mathbf{j})$ Amba's speed = $\sqrt{2.24^2 + 7.68^2}$ $= 8 \text{ m s}^{-1}$ |
| | Subtotal | | 1 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|-------|--|------|----------|--|
| 19(b) | Finds displacement when $t = 4$ using $\mathbf{s} = \mathbf{ut}$ PI by correct position vector | 1.1b | B1 | $\mathbf{s} = 4 \times \begin{bmatrix} 2.24 \\ 7.68 \end{bmatrix} = \begin{bmatrix} 8.96 \\ 30.72 \end{bmatrix}$ $\mathbf{r} = \begin{bmatrix} 2 \\ -7 \end{bmatrix} + \begin{bmatrix} 8.96 \\ 30.72 \end{bmatrix}$ $\mathbf{r} = \begin{bmatrix} 10.96 \\ 23.72 \end{bmatrix} \text{ metres}$ |
| | Uses their displacement with $\begin{bmatrix} 2 \\ -7 \end{bmatrix}$ to find Amba's position vector | 3.4 | M1 | |
| | Obtains $\begin{bmatrix} 10.96 \\ 23.72 \end{bmatrix}$ metres OE Condone missing or incorrect units | 1.1b | A1 | |
| | Subtotal | | 3 | |

| Q | Marking instructions | AO | Marks | Typical solution |
|-------|--|------|-----------|--|
| 19(c) | Finds Jo's speed PI by obtaining 40 m for Jo's distance | 1.1a | M1 | <p>Jo's speed = $\sqrt{2.8^2 + 9.6^2} = 10 \text{ m s}^{-1}$</p> <p>Jo's distance = $4 \times 10 = 40 \text{ m}$</p> <p>Amba's distance = $8 \times 4 = 32 \text{ m}$</p>  <p>$\sqrt{5^2 - \left[\frac{40-32}{2}\right]^2} = 3 \text{ m}$</p> |
| | Finds the distance travelled by Jo | 1.1b | A1 | |
| | Finds distance travelled over 4 seconds by Amba | 1.1b | B1 | |
| | Uses appropriate method to determine required distance | 3.1b | M1 | |
| | Obtains distance = 3 metres Must state units | 3.2a | A1 | |
| | Subtotal | | 5 | |
| | Question Total | | 10 | |